PROCEEDINGS

OF

THE PHYSICAL SOCIETY

OF LONDON.

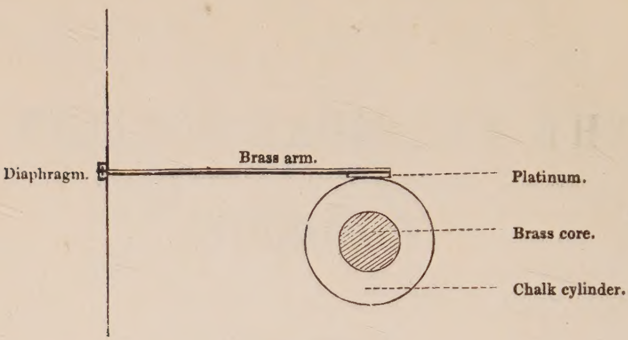
AUGUST 1880.

I. *The Influence of Friction upon the Generation of a Voltaic Current. By* SHELFORD BIDWELL, *M.A.**

IN 'Nature' of March 4th Prof. Barrett calls attention to the fact (which I believe had been observed before) that the electro-motograph, or receiving-instrument of Edison's loud-speaking telephone, is capable of generating an electric current. He considers that we have here a new source of electricity, the current generated being due to the motion of a chalk cylinder under a metallic surface. His chalk, he adds in a note, had been impregnated some months before with a solution of phosphate of soda, but was, when used, practically dry.

The construction of the motograph is essentially as follows:—One end of an arm of brass is joined to the diaphragm of the instrument; the other end, which is faced with platinum, is pressed by a spring against the upper surface of a horizontal chalk cylinder. This cylinder is mounted on a thick brass core or spindle, and can be made to rotate and rub against the platinum on the arm by turning a winch-handle. The cylinder is impregnated with certain chemical substances, of which I believe caustic potash is always one, though it is not mentioned by Prof. Barrett.

* Read March 13, 1880.



It will be seen that there are here two metals, brass and platinum, connected by a medium containing caustic potash; and the arrangement looks so much like a voltaic couple that I was not surprised, on connecting the brass and platinum to a galvanometer, to find a small + current flowing from the platinum-faced arm. The cylinder was not rotated, and had not been moistened for some months. On turning the handle the galvanometer at once indicated a much stronger + current passing from the platinum. The effect of the friction was to greatly increase the strength of the current.

I then faced the brass arm with zinc instead of platinum. On connecting the instrument to the galvanometer a small deflection was again observed; but this time the + current flowed, as I expected, not from the zinc-faced arm, but from the brass core of the cylinder. The substitution of the zinc for the platinum had the effect of reversing the current, brass being positive to platinum and negative to zinc. In this case too, rotation of the cylinder largely increased the current.

I now laid aside the motograph, and connected to the two wires of the galvanometer a sheet of brass and a sheet of platinum. Between these metallic plates I placed a thin slice of dry chalk; the galvanometer indicated nothing. I rubbed the metals successively against the chalk; still the galvanometer remained motionless. For the pure chalk I substituted a chalk plate which had been soaked in a saturated solution of phosphate of soda and thoroughly dried; again there was no result. Neither was there when I used dry blotting-paper which had been saturated with phosphate of soda. But with

a piece of blotting-paper which had been saturated with a solution of caustic potash and made as dry as possible, the results were just the same as with the electro-motograph: brass and platinum gave a + current from the platinum; brass and zinc gave a + current from the brass; and in both cases the current was much increased by rubbing.

This experiment was repeated with the following pairs of metals—brass and platinum, zinc and lead, zinc and copper, lead and copper, tin and copper, zinc and tin—covering one of every pair of metals successively with a wet cloth and rubbing the one so covered with the other. In every case the friction seemed to considerably increase the current which was generated on mere contact. If, for instance, a piece of lead is covered with a wet cloth and a piece of copper is pressed upon it, a + current will of course flow from the copper. On rubbing, this current is very greatly increased. If the copper be the metal covered, and the lead rubbed against it, the current from the copper will again be greatly increased, but apparently not quite so much so as in the former case. And I believe that this difference in the effect produced according to the metal covered, occurs in the case of all the other pairs of metals which I tried. In some cases it is very notable; in others it is small, and extremely difficult to detect without a machine for producing uniform friction. But, as far as I can judge, the effect on the current is always greatest when the positive element is covered and the negative element exposed to friction.

It seems, therefore, that Prof. Barrett's experiment is only an illustration of the effect of friction on one of the elements of a voltaic couple in increasing the current. This effect is, however, so very remarkable that I was induced to make further experiments.

The most curious result at which I arrived is this:—Take two plates of the same metal, cut from the same sheet, and connect them with a galvanometer; cover one of them with a wet cloth and bring the other down upon it. If the two pieces of metal are in the same physical condition there will be no material deflection of the galvanometer. Now rub the covered metal with the bare metal. As long as the rubbing continues, the galvanometer indicates a current of electricity,

which ceases as soon as the rubbing is stopped. And this current invariably flows from the plate which is covered by the wet cloth.

I have made the experiment with plates of tin, lead, copper, brass, and zinc, with the same result in every case.

I do not suggest an explanation of the above-mentioned phenomena. When the two plates of a single metal are used, it appears that friction renders the one rubbed relatively electro-positive. But in the case of two metals, friction seems generally to have a greater effect upon the negative than the positive element, and it makes the negative element not more positive but more negative. Any possible explanation that occurs to me of the one case is inconsistent with the other.

Certain speculations led me to try the effect of passing a battery-current through a pair of metallic plates, separated by a piece of wet rag or paper, while they were being rubbed together; and I found that when the current passed from the covered to the uncovered plate, a remarkable diminution in the friction occurred. A current in the opposite direction produced no such result. I exhibit a little apparatus for rendering this effect visible to a large audience.

This experiment seems to show conclusively that the generally received theory of electrolytic action is sufficient to explain the phenomena presented by the electro-motograph; for it is difficult to conceive any other possible effect of the current than the liberation of hydrogen on the surface of the plate which is connected with the negative pole of the battery, the layer of hydrogen having the effect of diminishing the friction.

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II. *On some Effects of Vibratory Motion in Fluids; on the Attraction due to the Flow of Liquids from an Expanded Orifice; and Laboratory Notes.* *By* R. H. Ridout.

On some Effects of Vibratory Motion in Fluids.

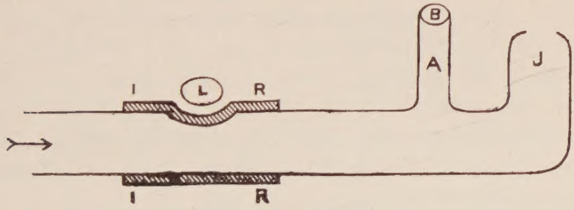
On causing a stream of coal-gas to bubble through a liquid contained in a flask, it may by careful adjustment be made to issue from a suitable jet in an oscillatory manner, in the plane which would contain a sensitive flame issuing from

the same jet. Thus a flame may be sensitive when the disturbance is imparted from within.

An analogous experiment with water was attempted as follows:—An electromagnetic engine was made to oscillate a lever, pressing against an india-rubber tube, conveying water from a height of 14 feet to a partially closed tube, such as would emit a sensitive flame under a moderate pressure. The water issued in a pulsatory manner, but showed no new form.

On comparing the two experiments, it will be seen that the bubble of gas, in passing through the liquid, is gradually increasing in volume, and when it reaches the surface, probably expands to a size it could not permanently retain. Such is not possible with water; for though its elasticity is greater than that of air, the range through which it acts (which may be called its amplitution) is much less. To provide for this amplitution, an air-chamber was introduced between the lever and the jet (fig. 1). The water then issued in exactly the

Fig. 1 (section).



L = Lever.
I R = India-rubber tube.
A = Air-chamber.
B = Air-bubble.
J = Jet.

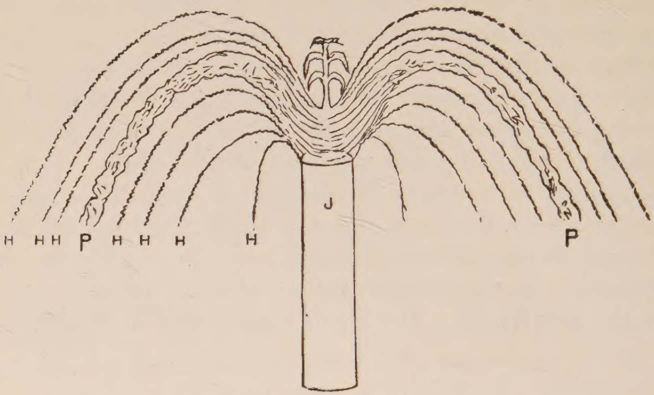
same manner as an excited sensitive flame, breaking into two streams lying in *one* plane. The motion of the lever upon the flowing water produces in the latter a sinuous wave; and as the troughs and crests reach opposite sides of the jet, they take alternatively opposite directions, forming two complete streams with about 400 vibrations per second. On substituting a straight tube for the jet, figures of great beauty were obtained.

Fig. 2 shows a simple wave free from harmonics. The phase of the wave relatively to the lever's motion is so strongly marked, that it is not destroyed by passing through tortuous

ubes 2 feet long. When the lever presses the tube at any but a right angle, the figures are more complex and show a tendency to rotation.

A wave with harmonics produces minor streams (fig. 3) in

Figs. 2 and 3.



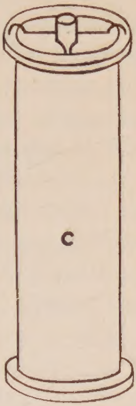
P P. Figure resulting from primary wave.
H H &c. Harmonics. J. Straight tube.

addition to the primary one. I have shown in 'Nature' (Oct. 3, 1878) that a flame is capable of producing harmonics of its primary note; and this makes the analogy complete between gaseous and liquid columns, in their behaviour to sensible vibratory motion.

The experiment analogous to the ordinary sensitive flame was attempted by immersing a sensitive jet in a vessel of water and passing a coloured liquid. The jet always "roared" when a higher pressure than $\frac{1}{2}$ in. water was used. Near the critical point I could not produce any marked effect by applying to the water a 4-inch tuning-fork. Possibly larger apparatus would give better results. While working with this, I was struck with the great depth to which a stream of the liquid will descend under a very small pressure from a non-sensitive jet, made by contracting a $\frac{5}{8}$-inch test-tube to about $\frac{1}{16}$ inch, and cutting off some distance below the contraction. When a liquid flows through this with a fall of $\frac{1}{4}$ inch into a tall cylinder of the same liquid, it continues in an unbroken line for 8 inches or more, having the diameter of the smallest part. Recalling a deduction of Froude's, that a liquid flowing through

a tortuous tube without friction has no tendency to straighten it, I mounted the jet on pivots, and, by giving it an oscillatory motion, found that the stream travelled the liquid as a perfect wave, showing the deduction to be experimentally true. This apparatus (fig. 4), if mounted on the board carrying the vibrating lever and water-jet, gives a graphic representation of the wave which is producing the figures (figs. 2 and 3), and shows the presence or absence of harmonics.

Fig. 4.

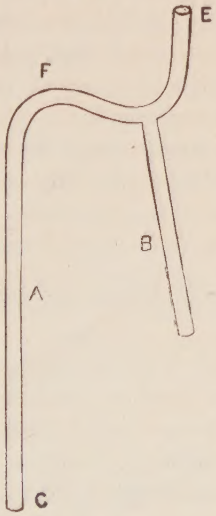


By using in the glass cylinder a weak solution of oxalic and sulphuric acids, and supplying the jets with a solution of sulphuric acid and permanganate of potash of exactly the same density, the waves are extinguished after a few seconds, while the liquid remains quite clear.

It will be seen that the streams (figs. 2 and 3) soon break into beads. On receiving these in water, they manifest the spheroidal state in a high degree.

When the bubble of air in the air-chamber (fig. 1) is above a certain size it splits into several lesser ones; and these on entering the current have a less velocity than the water towards the jet. Fig. 5 shows how this effect may be magnified. In a glass tube of ¼-inch bore, the limb A is about 1 foot long and closed at C. An india-rubber tube is attached at E, and bent back at a small angle in order to vibrate, as in the Bunsen pump. B is the waste-pipe. A bubble of air, the size of a rape-seed, is placed in the bend at F, A being full of water. When the current of water through E causes the whole to vibrate, the air-bubble detaches itself from the tube, takes the shape of a string of four or five beads, and sinks through the water to C, where it remains as long as the vibration continues. Attempts were made to substitute sphericles of glass,

Fig. 5.



shellac, wax, &c., each made a little lighter than water, but without success. If the result were due to the passage of vibration being more rapid in water than air, then globules of mercury, bromine, and bisulphide of carbon, all of which have a greater velocity for sound than water, should either float or show some tendency to do so. A great number of experiments, under varying conditions, failed to show that they were in any way affected.

As an air-bubble sinks as rapidly as a grain of sand would do, I tried hydrogen in place of air, and mercury in place of water in the tube A. When the hydrogen reaches the mercury it rests a little, and then suddenly sinks with the same apparent velocity as the air in water.

If in these experiments the bubble is large it does not descend; and this points to the close analogy with the experiments of Guthrie (Proc. Roy. Soc. 1870, pp. 35–41), who found that the vibration of a solid body caused the "approach" of a piece of cardboard from a considerable distance. We might say, conventionally, that the attraction was less between the air and the sounding body than the cardboard and the latter. Now in the tube A, C being a node, any particle near C is relatively at rest to one nearer E, and the attraction between them is greater than either for the bubble. Hence the particle at C tends to move towards E, or, what is the same thing, the bubble moves to C. When the bubble is large it is still repelled, but it leaves no way for the approach of the attracted water. A great number of small bubbles may be successively sunk till a large one results at C, showing that the conditions of the experiment only proscribe the motion of the large one. By inverting fig. 5 it may be used to produce the water-figures; but they can seldom be obtained free from harmonics by such means.

On the Attraction due to the Flow of Liquids from an Expanded Orifice.

M. Lacoutre has shown in *Les Mondes* for May 10, 1866, that when a liquid flows perpendicularly from a cylindrical tube upon a flat disk, its motion is arrested till the disk is brought to within half a diameter of the tube (but varying with pressure), when the outflow is greater than if the disk be

entirely removed. I find that if the tube terminate in a similar disk, attraction ensues at the position of greatest outflow.

A glass filter funnel containing a light cone with a weight dependent from its apex, or a tube terminating in a hemispherical cup with a light ball, have more stability than the disks, and answer equally well. In all cases the apparatus may be immersed in liquid and the attraction continued. The explanation usually accepted of the corresponding pneumatic experiment of M. Clément Desormes, is based on the assumption that the velocity of the fluid gradually decreases as it approaches the edges of the disks. To determine how far this was generally true, I took an outer cone A of 4 inches diameter at base with angle of 60° at apex, and an inner cone B upon same base with angle of 120°. For the velocity of the liquid to diminish between the apex of B and base of A, the cones must be separated nearly $\frac{1}{4}$ inch. Using water at varying pressure, in no case did attraction take place at more than $\frac{1}{10}$ in., where the velocity must *increase* towards the outlet. I find also that the presence of the attracted cone, ball, &c. diminishes the pressure in the supply-pipe, as shown by a water-guage attached to the latter. Hence the presence of the cone &c. facilitates the efflux.

Laboratory Notes.

Apparatus for showing Electrolysis of Water.

A glass bolthead of 30 or 40 oz. capacity is stopped with an india-rubber cork carrying two glass tubes, which contain hermetically sealed platinum wires, projecting an inch at the inner end and terminating in binding-screws at the other. The vessel is filled one fifth full of acidulated water, boiled, and the stopper inserted to cause a vacuum when cold. On connecting with two "Grove" cells, the bubbles of gas so expand as to make the whole liquid appear to boil. With either a single "Grove," "Bunsen," "bichromate," or "Leclanché" cell continuous decomposition may be obtained. When sufficient gas has collected to impair the vacuum it may be restored by boiling.

Experiment showing Cohesion in Liquids.

A shallow tray 6″ × 2″, open at one end and lipped, is supported on three levelling-screws, the lipped end being *slightly*

higher than the other. A quantity of mercury placed in the tray falls to the lower end; but if now a little more be added to make it flow over the lip, the cohesion is such as to enable the descending stream to drag the remainder up the inclined plane. Water gives similar results; but, from the difficulty of getting a surface which will long remain unwetted, the results are not so satisfactory.

Production of a Musical Note in a Continuous Tube.

In most wind instruments the sound results either from the movement of a solid body, or the air has the choice of two directions, which it alternately takes. I find, however, that it is possible to produce a good note from a tube $\frac{1}{4}$ to $\frac{5}{8}$ inch in diameter, and from 6 inches to a foot long, and having a part of it contracted smoothly and evenly to about a fourth of its diameter, by blowing through it. If the tube be bent upon itself at the point of contraction, the sounds are more readily obtained, though not of greater intensity.

Apparatus for showing Absorption of Heat on Liquefaction of Solids.

In a differential air-thermometer, the usual flasks are replaced by others which have had their bottoms softened, and then introverted to form a cup or basin. In this latter water is placed, and the solid then added. Any change in the liquid's temperature is at once communicated to the air-space round the cup.

Experiment showing the Expansion of Glass by Heat, and its low Conducting-power.

A glass tube of $\frac{1}{8}$-inch bore and 18 inches long is bent into the form of a violet-leaf—its free ends forming the apex, and leaving an interval in which a coin may be held by the elasticity of the glass. When heat is applied to the convex side of the middle bend, the limbs open and the coin falls.

III. *Preliminary Notes on* Mr. Hall's *Recent Discovery.* *By* H. A. Rowland*.

The recent discovery by Mr. Hall† of a new action of magnetism on electric currents opens a wide field for the

* From the American Journal of Mathematics.

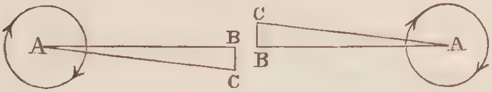
† Phil. Mag. [5] vol. ix. p. 225.

mathematician, seeing that we must now regard most of the equations which we have hitherto used in electromagnetism as only approximate, and as applying only to some ideal substance which may or may not exist in nature, but which certainly does not include the ordinary metals. But as the effect is very small, probably it will always be treated as a correction to the ordinary equations.

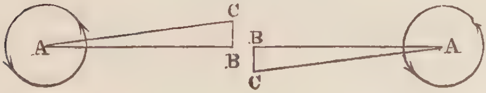
The facts of the case seem to be as follows, as nearly as they have yet been determined:—Whenever a substance transmitting an electric current is placed in a magnetic field, besides the ordinary electromotive force in the medium, we now have another acting at right angles to the current and to the magnetic lines of force. Whether there may not be also an electromotive force in the direction of the current has not yet been determined with accuracy; but it has been proved, within the limits of accuracy of the experiment, that no electromotive force exists in the direction of the lines of magnetic force. This electromotive force in a given medium is proportional to the strength of the current and to the magnetic intensity, and is reversed when either the primary current or the magnetism is reversed. It has also been lately found that the direction is different in iron from what it is in gold or silver.

To analyze the phenomenon in gold, let us suppose that the line A B represents the original current at the point A, and that B C is the new effect. The magnetic pole is supposed to be either above or below the paper, as the case may be. The line A C will represent the final resultant electromotive force at the point A. The circle with arrow represents the direction in which the current is rotated by the magnetism.

North Pole above.



North Pole below.



It is seen that all these effects are such as would happen were the electric current to be rotated in a fixed direction with respect to the lines of magnetic force, and to an amount depending only on the magnetic force and not on the current. This fact seems to point immediately to that other very important case of rotation, namely the rotation of the plane of polarization of light. For, by Maxwell's theory, light is an electrical phenomenon, and consists of waves of electrical displacement, the currents of displacement being at right angles to the direction of propagation of the light. If the action we are now considering takes place in dielectrics, which point Mr. Hall is now investigating, the rotation of the plane of polarization of light is explained.

I give the following very imperfect theory at this stage of the paper, hoping to finally give a more perfect one either in this paper or a later one.

Let $\mathfrak{H}$ be the intensity of the magnetic field, and let E be the original electromotive force at any point, and let $\mathfrak{c}$ be a constant for the given medium. Then the new electromotive force E′ will be

$$E' = \mathfrak{c}\mathfrak{H}E,$$

and the final electromotive force will be rotated through an angle which will be very nearly equal to $\mathfrak{c}\mathfrak{H}$. As the wave progresses through the medium, each time it (the electromotive force) is reversed it will be rotated through this angle; so that the total rotation will be this quantity multiplied by the number of waves. If λ is the wave-length in air, and i is the index of refraction, and c is the length of medium, then the number of waves will be $\frac{ci}{\lambda}$, and the total rotation

$$\theta = c\mathfrak{c}\mathfrak{H}\frac{i}{\lambda}.$$

The direction of rotation is the same in diamagnetic and ferromagnetic bodies as we find by experiment, being different in the two ; for it is well known that the rotation of the plane of polarization is opposite in the two media, and Mr. Hall now finds *his* effect to be opposite in the two media. This result I anticipated from this theory of the magnetic rotation of light.

But the formula makes the rotation inversely proportional

to the wave-length, whereas we find it more nearly as the square or cube. This I consider to be a defect due to the imperfect theory; and it would possibly disappear from the complete dynamical theory. But the formula at least makes the rotation increase as the wave-length decreases, which is according to experiment. Should an exact formula be finally obtained, it seems to me that it would constitute a very important link in the proof of Maxwell's theory of light, and, together with a very exact measure of the ratio of the electromagnetic to the electrostatic units of electricity which we made here last year, will raise the theory almost to a demonstrated fact. The determination of the ratio will be published shortly; but I may say here that the final result will not vary much, when all the corrections have been applied, from 299,700,000 metres per second; and this is almost exactly the velocity of light. We cannot but lament that the great author of this modern theory of light is not now here to work up this new confirmation of his theory, and that it is left for so much weaker hands.

But before we can say definitely that this action explains the rotation of the plane of polarization of light, the action must be extended to dielectrics, and it must be proved that the lines of electrostatic action are rotated around the lines of force as well as the electric currents. Mr. Hall is about to try an experiment of this nature.

I am now writing the full mathematical theory of the new action, and hope to there consider the full consequences of the new discovery.

Addition.—I have now worked out the complete theory of the rotation of the plane of polarization of light, on the assumption that the displacement currents are rotated as well as the conducted currents. The result is very satisfactory, and makes the rotation proportional to $\frac{i^{\cdot 5}}{\lambda^2}$, which agrees very perfectly with observation. The amount of rotation calculated for gold is also very nearly what is found in some of the substances which rotate the light the least. Hence it seems to me that we have very strong ground for supposing the two phenomena to be the same.

III. *Complete Theory of the Bifilar Magnetometer and new Methods for the Determination of the Absolute Horizontal Intensity of the Earth's Magnetism, as well as of the Temperature and Induction-coefficients of Magnets.* *By* H. Wild.

The above is the title of a communication made to the Imperial Academy of Sciences of St. Petersburg on the 15th (27th) January, 1880 (*Bull. Acad. Impér. Sci. St. Pétersburg, Mélanges Phys. et Chim.* xi.), as an abstract of a more detailed paper to be shortly laid before the Academy.

The starting-point of the paper is the observation that the indications of the bifilar magnetometer, even when all causes of disturbance are excluded, differ from the results of direct measures of horizontal intensity to a far greater extent than can be accounted for by errors of observation. The author traces the cause of these discrepancies in the twisting of the suspending fibres, of which no account is taken in the ordinary theory of the bifilar magnetometer, according to which the moment of the directive couple of the suspension is ascribed simply to the weight of the magnet and its stirrup. The effect which this cause is capable of producing may either be calculated from the dimensions and coefficient of torsion-elasticity of the fibres, or it may be determined experimentally upon the bifilar instrument itself by observing the period of vibration before and after a known change has been made in the moment of inertia. Concordant determinations by both methods indicated that in the case of a bifilar suspended by brass wires 4 metres long, the couple due to torsion amounted to 1 per cent. of that due to the suspended weight. In the bifilar magnetometer of the St.-Petersburg magnetograph, which was suspended by steel wires only 0·3 metre long, the effect of torsion was as much as about 7 per cent. of the gravitation-couple. On the other hand, it amounted to only 0·3 per cent. in a bifilar of the Pawlowsk Observatory, this instrument having cocoon fibres 1·2 metre long. From the consideration of the conditions which determine the magnitude of the torsion and gravitation-couples respectively, Prof. Wild concludes that it is not probable that the proportion between them can be reduced much below that which exists in the last-mentioned instrument, namely 0·3 per cent., and hence

that the correction for torsion cannot be of negligible amount in accurate observations.

When this correction is applied, as well as that for induction, the author finds that the differences between the results of calculation and experiment disappear, and that very nearly identical values of the characteristic magnitude of the bifilar magnetometer, namely the so-called "torsion-angle" z, are obtained by both methods. The reader may be reminded that this angle z is the angle which a vertical plane passing through the upper ends of the suspending fibres makes with a vertical plane through their lower ends when the magnet is perpendicular to the magnetic meridian. If D be the moment of the directive couple due to the suspension, H the earth's horizontal intensity, and M the moment of the magnet, the angle z is defined by the equation

$$D \sin z = HM.$$

Its magnitude may be found by direct experiment; or it may be calculated from the periods of vibration of the magnet observed when its position of equilibrium is in the magnetic meridian and its north-seeking pole is towards magnetic north and its south-seeking towards magnetic south respectively. If t_1 and t_2 are the periods in the two cases,

$$\sin z = \frac{t_2^2 - t_1^2}{t_2^2 + t_1^2}.$$

The author observes, in the next place, that the direct determination of z, together with the observation of the periods t_1, t_2, t_3 (the last being the period of vibration when the position of equilibrium of the magnet is perpendicular to the meridian), affords an excellent method for determining separately the two induction-coefficients of the magnet, namely the coefficient of increase and that of decrease of the magnetic moment. The advantages of this method are chiefly that it does not depend upon the mutual action of two magnets, which can be only approximately calculated, and that it requires the use of only one instrument. He also remarks that these advantages apply also to the method indicated by him in a previous publication for the determination of the temperature-coefficients of magnets, and that he has assured himself by actual trial of the precision of this method.

The remainder of the paper is devoted to the description of a process for determining in absolute measure the earth's horizontal magnetic intensity by means of the bifilar magnetometer, and to the statement of the formulæ required for the complete reduction of the results so as to take account of all corrections. The principles involved in this process may be thus indicated:—It is divisible, like Gauss's process, into a method for determining (*a*) the product of the horizontal intensity H into the moment M of a particular magnet, and (*b*) the ratio of the same two quantities. To determine the *product*, the magnet is suspended in the bifilar instrument, and the torsion-angle z is observed which is required to set the axis of the magnet perpendicular to the meridian. This operation gives the equation

$$D \sin z = HM.$$

The *ratio* of H to M is found by suspending another magnet of moment M′ in the bifilar, placing the first magnet with its axis in the magnetic meridian through the centre of suspension of the instrument, and determining the torsion-angle z_1, needed to set the suspended magnet perpendicular to the meridian when the north-seeking pole of the first magnet (moment M) is towards magnetic north, and also the corresponding angle z_2, when the north-seeking pole of the first magnet is towards magnetic south. These observations give, subject to the proper corrections, the equations

$$D' \sin z_1 = \left(H + \frac{2M}{r^3}\right) M',$$

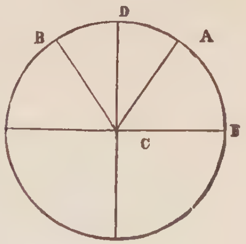
$$D' \sin z_2 = \left(H - \frac{2M}{r^3}\right) M';$$

from which the ratio in question is easily obtained in terms of z_1, z_2, and the distance r between the centres of the magnets. The horizontal intensity is then given by an expression which, when the corrections (for which we must refer to the original paper) are omitted, becomes

$$H^2 = \frac{2D \sin z}{r^3} \cdot \frac{\sin z_1 + \sin z_2}{\sin z_1 - \sin z_2}.$$

V. *On a simple Form of Saccharimeter.* *By* J. H. POYNTING, *Fellow of Trinity College, Cambridge, Professor of Physics in Mason's College, Birmingham.*

THE general principle of the modification of the saccharimeter which I shall describe in this paper is well known, and has already been applied in the construction of several standard instruments, such as Jellett's and Laurent's. This principle consists in altering the pencil of rays proceeding from the polarizer in such a way that, instead of the whole pencil having the same plane of polarization, the planes of the two halves are slightly inclined to each other. The analyzer is therefore not able to darken the whole field of view at once. In one position of the analyzer the one half of the field is quite dark; in another position, slightly different, the other half is dark; while when the analyzer is halfway between these two positions, the two halves of the field are equally illuminated. This will be seen from the accompanying figure.



Let C A be the trace of the plane of polarization of the right half of the pencil, and C B that of the other half. Let C D bisect A C B. Then, if C E represent the plane of polarization of the light which alone the analyzer will allow to pass, when the analyzer is turned so that C E is perpendicular to C A the right-hand side of the field is dark. When C E is perpendicular to C B the right-hand is partially illuminated (as C A has a component along C E), while the left-hand is dark. Halfway between these positions, when C E is perpendicular to C D, both sides appear equally illuminated. The analyzer being turned round till this equality of illumination is obtained, its position is noted on the attached circle. When an active substance is now inserted in the path of the rays, the planes C A, C B are both rotated through the same angle, and the analyzer has to be rotated through this angle to give the equal illumination once more. The circle again being read, the difference of readings gives the rotation due to the interposed substance.

In Jellett's saccharimeter the inclination of the planes of polarization of the two halves of the field is obtained by interposing a prism of Iceland spar. This is formed by cutting a rhomb nearly parallel to its optic axis, reversing one of the pieces, and then cementing the two together again with the plane of separation bisecting the pencil of rays.

In Laurent's instrument, for which homogeneous light is used, half the pencil is passed through a plate of quartz cut with its axis in the surface and parallel to its edge, the thickness being such that the extraordinary is retarded half a wave-length behind the ordinary. On emergence the directions of vibration in the two parts of the pencil, one of which has traversed the quartz, are equally inclined to the edge of the crystal. The inclination of the two to each other can be very easily altered by simply turning the polarizer.

The following arrangement is in place of the Iceland spar in Jellett's instrument, and of the quartz plate in Laurent's. It seems to be somewhat simpler, and gives fairly good results.

A circular plate of quartz cut perpendicular to the axis is divided along a diameter, and one half slightly reduced in thickness. The two halves are then reunited and interposed in the path of the pencil and at right angles to its direction. Since one half of the pencil passes through a slightly greater thickness of quartz, its plane of polarization is slightly more rotated than that of the other half; and the pencil therefore emerges with the planes of polarization of its two halves slightly inclined to each other. It is of course always necessary to use homogeneous light to avoid dispersion.

Mr. Glazebrook has very kindly given me the following numbers, which are taken at random from a large number of sets of readings he has obtained for the electromagnetic rotation of certain solutions of NaCl in water; the difference of thickness of the two plates being ·1 mm., and the inclination of the planes of polarization being therefore about 2° for the sodium-light used. The circle to which the analyzer was attached reads to 3′; but the vernier divisions can easily be further subdivided by eye.

	Circle-readings.
I. Current direct.	23° 45′
	23 46
	23 45
	23 45
Current reversed.	21 36
	21 34
	21 39
II. Current direct.	23 15
	23 16
	23 18
Current reversed.	22 19
	22 19
	22 20
III. Current direct.	23 30
	23 30
	23 28
Current reversed.	21 45
	21 47
	21 48

In order to vary the inclination of the two planes of polarization to each other, one of the halves of the quartz plate might be arranged like a Babinet's compensator, so that the difference of the two might be varied at will. The chief objection to the method seems to be that the quartz plate has to be adjusted very exactly perpendicular to the axis of the pencil.

A still simpler arrangement, which has yet only been tried in a somewhat rough form, consists in a cell containing some active liquid, say sugar solution. This cell is interposed in the path of the pencil; and in it is inserted a piece of plate glass several millims. thick, arranged so that one half the pencil passes through it. This half therefore passes through a less thickness of the active substance than the other half, and is less rotated. The two then emerge as before, having their planes of polarization slightly inclined to each other. This inclination, and consequently the sensitiveness of the instrument can be varied either by varying the strength of the active

solution, or the thickness of the plate of glass inserted in the cell.

This arrangement, as far as it has been tested, gives as good results as the previous one, while it is much more easily constructed and adjusted.

VI. *The Vibrations of a Film in reference to the Phoneidoscope.* *By* WALTER BAILY, *M.A.**

[Plate I.]

THE object of this paper is to consider the superposition of several systems of waves in a plane film, when the vibrations are perpendicular to the film, and the wave-lengths of all the systems are the same, with the view of discovering (1) what combinations of systems give simple results in an infinite film, (2) which of these combinations can exist in a finite film, and (3) which of the latter can account for appearances presented in the phoneidoscope.

Let us examine the case of a plane uniform film of infinite extent traversed by three systems of waves with straight fronts, the vibrations being perpendicular to the film, and the wave-length of each system being the same.

Draw (fig. 1) Ac, Bc, Cc meeting in c; and let $\angle BcC=2\alpha$; $\angle CcA = 2\beta$; $\angle AcB = 2\gamma$. Take $cB = cC = \lambda$. Draw $QcR \perp Ac$; $RBP \perp Bc$; $PCQ \perp Cc$. Join Pc, and draw Qb and Rb, bisecting $\angle\angle PQR$ and QRP. Draw $ca \parallel Qb$, $cm \perp Rb$, $an \perp Rc$. It may be easily shown that $\angle cab=\alpha$, $\angle abc=\beta$, $\angle bca=\gamma$;

$$cm=\frac{\lambda}{2\sin\gamma}, \quad an=\frac{\lambda\cos\beta}{2\sin\gamma\sin\alpha}, \quad ca=\frac{\lambda}{2\sin\gamma\sin\alpha}.$$

Let λ be the wave-length; Ac, Bc, Cc the directions of the three systems of waves; h, k, l their amplitudes; and d, e, f their phases at some point. The values of $(e-f)$, $(f-d)$, $(d-e)$ are independent of time, and depend only on the position of the point chosen. QR, RP, PQ are wave-fronts; for they are $\perp$ to the directions of the waves.

Consider the line Pc. Every point on this line is equidis-

* Read June 12.

tant from the wave-fronts RP and PQ, so that $(e-f)$ is constant along Pc. Similarly $(d-e)$ is constant along ab, and $(f-d)$ is constant along Rb; and as we may shift the point c, it follows that along any lines $\parallel$ Pb, Qb, and Rb we have $(e-f)$, $(d-e)$, and $(f-d)$ constant respectively. Hence $(f-d)$ is constant along ca.

The value of e at c differs by 2π from its value along PQ, since B$c=\lambda$. Hence the value of $(d-e)$ at c differs by 2π from its value along ab. Similarly the values of $(f-d)$ at b, and of $(e-f)$ at a differ by 2π from the corresponding values along ca and bc.

Now the wave-fronts Rc, cB, BR will reach a simultaneously after crossing distances $= an$. Hence the difference of phase of the whole vibration at c and that at a

$$=\frac{an}{\lambda}2\pi=\frac{\pi\cos\beta}{\sin\gamma\sin\alpha}.$$

Draw (fig. 2) a series of straight lines $\parallel ab$ at distances $\frac{\lambda}{2\sin\gamma}$ apart. Along each of these lines $(d-e)$ is constant; and its value changes by 2π in passing from any line to the next. Draw another series $\parallel bc$ at distances $\frac{\lambda}{2\sin\alpha}$ apart. These lines will have similar properties with respect to $(e-f)$. Straight lines drawn through the intersections of these two series will form a series $\parallel ca$, at distances $\frac{\lambda}{2\sin\beta}$ apart, and having similar properties with respect to $(f-d)$. At all the intersections $(d-e)$, $(e-f)$, $(f-d)$ have the same values respectively; and therefore at all the intersections the amplitude of the vibration of the film is the same.

We can find a line such that $(d-e)$ is a multiple of 2π, and another line such that $(e-f)$ is a multiple of 2π. At their intersection $(f-d)$ will be also a multiple of 2π. Such a point is a ventral segment. Suppose (fig. 2) the lines to be moved until one of the intersections lies on a ventral segment; then all the intersections are ventral segments.

Now in a Δ with ventral segments at its $\angle$s, let p, q, r be the distances of a point from the sides of the Δ. Let p', q', r' be the differences between the values of $(e-f)$, $(f-d)$, $(d-e)$

at the point and on the respective sides. Then

$$p' : 2\pi = p : \frac{\lambda}{2\sin\alpha};$$

hence

$$p = \frac{p'\lambda}{4\pi\sin\alpha}, \quad q = \frac{q'\lambda}{4\pi\sin\beta}, \quad r = \frac{r'\lambda}{4\pi\sin\gamma}.$$

The displacement of the film at the point at any time may be expressed as

$$h\cos(d+\tau) + k\cos(e+\tau) + l\cos(f+\tau),$$

where τ is proportional to the time after the given moment. The maximum displacement will be

$$\sqrt{\{h^2+k^2+l^2+2kl\cos p'+2lh\cos q'+2hk\cos r'\}};$$

that is,

$$\sqrt{\left\{4s^2-4kl\sin^2\frac{p'}{2}-4lh\sin^2\frac{q'}{2}-4hk\sin^2\frac{r'}{2}\right\}},$$

where

$$2s = h+k+l.$$

This expression vanishes when

$$\sin^2\frac{p'}{2} = \frac{s(s-h)}{kl}, \quad \sin^2\frac{q'}{2} = \frac{s(s-k)}{lh}, \quad \sin^2\frac{r'}{2} = \frac{s(s-l)}{hk}.$$

These values show that there is a node in each triangle the position of which depends on the relative amplitudes of the waves. If one of the amplitudes (say l) is very nearly equal to the sum of the other two, $(s-l)$ is very small, and therefore r' is very small, and consequently r is very small; the result is that the nodes lie in pairs very near together. When one amplitude equals the sum of the other two, the pairs of nodes coalesce; and when one amplitude is greater than the sum of the other two, there can be no node.

In the particular case in which the directions of the waves are inclined at angles of 120° to one another, and the amplitudes are equal, the triangles in fig. 2 become equilateral, with their sides $=\frac{2\lambda}{3}$, and the nodes are equidistant from the ventral segments. Also the difference of phase between successive angles of the triangles is 120°. In fig. 3 let the dots represent the nodes and the numbers the ventral segments.

Then all the *ones* move together, and so do all the *twos*, and also all the *threes*, the difference of phase between the different sets being 120°.

This last case may be investigated algebraically. Let $h =$ the amplitude of each wave, v the wave-velocity, t the time, x, y and r, θ the coordinates of a point in the film, and z the displacement at that point, at the time t. Take a ventral segment as origin, and the direction of one of the waves as axis of x. Then

$$\begin{aligned} h^{-1}z = &\cos\left[2\pi\lambda^{-1}\{vt - r\cos\theta\}\right] \\ &+ \cos\left[2\pi\lambda^{-1}\{vt - r\cos(\theta - 120°)\}\right] \\ &+ \cos\left[2\pi\lambda^{-1}\{vt - r\cos(\theta + 120°)\}\right] \\ = &\cos\{2\pi\lambda^{-1}(vt - x)\} \\ &+ 2\cos\left\{2\pi\lambda^{-1}\left(vt + \frac{x}{2}\right)\right\}\cos(\pi\lambda^{-1}y\sqrt{3}). \end{aligned}$$

When z is a maximum we must have

$$\cos\{2\pi\lambda^{-1}(vt - x)\} = 1;$$

and either

$$\cos\left\{2\pi\lambda^{-1}\left(vt + \frac{x}{2}\right)\right\} = 1, \text{ and } \cos(\pi\lambda^{-1}\sqrt{3}) = 1,$$

or

$$\cos\left\{2\pi\lambda^{-1}\left(vt + \frac{x}{2}\right)\right\} = -1, \text{ and } \cos(\pi\lambda^{-1}\sqrt{3}) = -1.$$

These conditions are satisfied by the sets of values given in the following table; l, m, n being any integers:—

$\lambda^{-1}vt$	$l - \frac{1}{3}$	$l - \frac{1}{3}$	l	l	$l + \frac{1}{3}$	$l + \frac{1}{3}$
$\lambda^{-1}x$	$2m + \frac{2}{3}$	$2m - \frac{1}{3}$	$2m$	$2m + 1$	$2m - \frac{2}{3}$	$2m + \frac{1}{3}$
$\lambda^{-1}y\sqrt{3}$	$2n$	$-2n$	$2n$	$-2n$	$2n$	$-2n$

Let Z be the amplitude of the vibration at any point, then Z is the maximum value of z with respect to t. Obtaining this we get

$$h^{-2}Z^2 = \sin^2(\pi\lambda^{-1}3x) + \{\cos(\pi\lambda^{-1}3x) + 2\cos(\pi\lambda^{-1}y\sqrt{3})\}^2.$$

At the nodes $Z = 0$, and therefore

$$\sin(\pi\lambda^{-1}3x) = 0,$$

and

$$\cos(\pi\lambda^{-1}3x) + 2\cos(\pi\lambda^{-1}y\sqrt{3}) = 0.$$

These conditions are satisfied when

$$\lambda^{-1}3x=2m \quad \text{and} \quad \lambda^{-1}y\sqrt{3}=2n\pm\tfrac{1}{3},$$

or

$$\lambda^{-1}3x=2m+1 \text{ and } \lambda^{-1}y\sqrt{3}=2n\pm\tfrac{2}{3}.$$

Putting $y=0$ in the above equations, we get

$$h^{-1}z=\cos\{2\pi\lambda^{-1}(vt-x)\}+2\cos\left\{2\pi\lambda^{-1}\left(vt+\frac{x}{2}\right)\right\},$$
$$h^{-2}Z^2=5+4\cos(\pi\lambda^{-1}3x).$$

The former of these equations gives a section of the film at time t, through a ventral segment in the direction of any one of the waves; and the latter gives a similar section of the surface which encloses the space within which this film vibrates. By putting $x=0$, we get

$$h^{-1}z=\cos(2\pi\lambda^{-1}vt)\{1+2\cos(\pi\lambda^{-1}y\sqrt{3})\},$$
$$h^{-2}Z^2=\{\cos(\pi\lambda^{-1}3x)+2\}^2;$$

and these equations give similar sections to the former ones, but in directions parallel to the fronts of the waves.

The next case we will examine is that of six waves of the same amplitude meeting each other in pairs, the directions of the pairs being inclined to one another at angles of 120°, with the condition that at some one point all the vibrations shall be in the same phase. These may be divided into two sets of three waves each; and the position of the ventral segments of the first set may be represented as in fig. 2. The position of the ventral segments of the second set may be represented by a similar figure, except that we should have to put 2 instead of 3, and 3 instead of 2, in numbering the ventral segments. In superposing the one figure on the other, we must make a ventral segment of the one figure coincide with a ventral segment of the same numeral of the other. Let a *one* of each figure coincide, then all the *ones* will coincide, and will indicate the points of maximum vibration of the film. On the other numerals the film will not have its maximum vibration, as one set of vibrations will partly destroy the other.

We can get a simple algebraic expression for the form of the film.

Divide the waves into two sets as before, and let z_1 be the displacement due to one set, z_2 that due to the other. Then

we have

$$h^{-1}z_1 = \cos\{2\pi\lambda^{-1}(vt-x)\} + 2\cos\left\{2\pi\lambda^{-1}\left(vt+\frac{x}{2}\right)\right\}\cos(\pi\lambda^{-1}y\surd 3).$$

By changing the sign of x and y we turn the whole figure through 180°, and so reverse the motions of the waves; hence we get

$$h^{-1}z_2 = \cos\{2\pi\lambda^{-1}(vt+x)\} + 2\cos\left\{2\pi\lambda^{-1}\left(vt-\frac{x}{2}\right)\right\}\cos(\pi\lambda^{-1}y\surd 3);$$

$$\begin{aligned}\therefore h^{-1}z &= h^{-1}z_1 + h^{-1}z_2 \\ &= \cos(2\pi\lambda^{-1}vt)\{2\cos(2\pi\lambda^{-1}x) \\ &\qquad + 4\cos(\pi\lambda^{-1}x)\cos(\pi\lambda^{-1}y\surd 3)\},\end{aligned}$$

$$h^{-1}Z = 2\cos(2\pi\lambda^{-1}x) + 4\cos(\pi\lambda^{-1}x)\cos(\pi\lambda^{-1}y\surd 3).$$

The results of this equation are represented in fig. 4. The large dots represent the points at which Z is at its maximum, viz. $6h$. They occur when

$$-\cos\pi\lambda^{-1}x = \cos\pi\lambda^{-1}y = \pm 1;$$

that is, when

$$x = 2m\lambda, \qquad y\surd 3 = 2n\lambda,$$

and when

$$x = (2m+1)\lambda, \quad y\surd 3 = (2n+1)\lambda.$$

The small dots represent points at which $Z = -3h$. Putting this value for Z, the equation becomes

$$0 = \sin^2(\pi\lambda^{-1}y\surd 3) + \{\cos(\pi\lambda^{-1}y\surd 3) + 2\cos(\pi\lambda^{-1}x)\}^2.$$

This is satisfied when

$$2\cos(\pi\lambda^{-1}x) = -\cos(\pi\lambda^{-1}y\surd 3) = \pm 1;$$

that is, when

$$y\surd 3 = 2n\lambda, \qquad x = (2m\pm\tfrac{2}{3})\lambda,$$

and when

$$y\surd 3 = (2n+1)\lambda, \quad x = (2m\pm\tfrac{1}{3})\lambda.$$

The dotted lines give the locus of points at which $Z = -2h$. Putting this value into the equation, we get

$$0 = \cos(\pi\lambda^{-1}x)\{\cos(\pi\lambda^{-1}x) + \cos(\pi\lambda^{-1}y\surd 3)\}.$$

This equation is satisfied when

$$x = (m+\tfrac{1}{2})\lambda,$$

and when

$$x \pm y\surd 3 = (2n+1)\lambda;$$

so that the locus consists of three sets of parallel straight lines.

The nodal lines are obtained by putting $Z=0$. The equation to them is

$$0=\cos(2\pi\lambda^{-1}x)+2\cos(\pi\lambda^{-1}x)\cos(\pi\lambda^{-1}y\sqrt{3}).$$

It is obvious from the loci already obtained, that these lines must be closed curves surrounding the points for which $Z=6h$; and that they must approximate to an hexagonal form, the greatest radii being towards the corners, and the least perpendicular to the sides of the hexagons formed by the locus of $Z=-2h$.

Putting $y=0$, we have

$$x=\frac{\lambda}{\pi}.\cos^{-1}.\frac{\sqrt{3}-1}{2}=\cdot 381\,.\,\lambda.$$

Putting $x=0$, we have

$$y=\frac{2\lambda}{3\sqrt{3}}\qquad\qquad=\cdot 385\,.\,\lambda.$$

These are the values of the greatest and least radii; and therefore the nodal lines are very nearly circles, with radii $=\cdot 383\,.\,\lambda$, and centres at the points for which $Z=6h$. The nodal lines are represented by the circles in fig. 4.

We will next consider the case of four waves meeting two and two, the angle between these directions being 2α, the amplitude of all the waves being the same, with the condition that at one point all the waves shall be in the same phase. We have

$$\begin{aligned}h^{-1}z= {} & \cos\left[2\pi\lambda^{-1}\{vt-r\cos(\theta-\alpha)\}\right] \\ & \qquad +\cos\left[2\pi\lambda^{-1}\{vt+r\cos(\theta-\alpha)\}\right] \\ & +\cos\left[2\pi\lambda^{-1}\{vt-r\cos(\theta+\alpha)\}\right] \\ & \qquad +\cos\left[2\pi\lambda^{-1}\{vt+r\cos(\theta+\alpha)\}\right] \\ = {} & 4\cos(2\pi\lambda^{-1}vt)\cos(2\pi\lambda^{-1}x\cos\alpha)\cos(2\pi\lambda^{-1}y\sin\alpha).\end{aligned}$$

The ventral segments occur when the quantities have the values given in the following table:—

$\lambda^{-1}vt$	l	l	$l+\frac{1}{2}$	$l+\frac{1}{2}$
$\lambda^{-1}x\cos\alpha$	m	$m+\frac{1}{2}$	$m+\frac{1}{2}$	m
$\lambda^{-1}y\sin\alpha$	n	$n+\frac{1}{2}$	n	$n+\frac{1}{2}$

The ventral segments are divided into two sets: all one set

vibrate together; and all the other set vibrate in the opposite direction.

The nodal lines are two sets of straight lines, whose equations are

$$4x \cos \alpha = (2m+1)\lambda$$

and

$$4y \sin \alpha = (2n+1)\lambda.$$

Hence the nodal lines divide the film into oblongs whose length and breadth are

$$\frac{\lambda}{2 \cos \alpha} \text{ and } \frac{\lambda}{2 \sin \alpha};$$

and a ventral segment lies in the centre of each oblong. When the directions of the waves are $\perp$ each other, $\alpha = 45°$, and the oblongs become squares. This form is shown in fig. 5.

When two waves of equal amplitude meet at an angle $= 2\alpha$, the equation is

$$\begin{aligned} h^{-1}z = \cos\left[2\pi\lambda^{-1}\left\{vt - r\cos(\theta - \alpha)\right\}\right] \\ + \cos\left[2\pi\lambda^{-1}\left\{vt - r\cos(\theta + \alpha)\right\}\right] \\ = 2\cos\left\{2\pi\lambda^{-1}(vt - x\cos\alpha)\right\} . \cos(2\pi\lambda^{-1}y\sin\alpha). \end{aligned}$$

The nodal lines are the straight lines

$$4y \sin \alpha = (2n+1)\lambda.$$

When $\alpha = 90°$, we get two waves meeting each other in the same direction. The equation becomes

$$h^{-1}z = 2\cos(2\pi\lambda^{-1}vt)\cos(2\pi\lambda^{-1}y).$$

Hitherto we have dealt only with infinite films; we will now consider what forms of vibration can be maintained in films of limited size, the waves undergoing reflection from the boundaries of the film.

When the film is an equilateral triangle, suppose a set of waves to be started having their fronts $\perp$ to one of the sides of the triangle. These waves will be reflected so as to have their fronts $\perp$ to another side, and again reflected so as to have their fronts $\perp$ to the remaining side, and by another reflection they will assume their first direction. If the wave-length is such that the time a wave takes to return to the same position is an integral number of wave-periods, we shall have the case of three sets of equal waves meeting each other at

angles of 120°. Fig. 8 shows the form of vibration when the wave-length is $\frac{3}{10}$ of the side of the triangle (no allowance being made for the change of phase at the reflections). The thin lines show the position of the maximum displacement due to each wave at one of the instants at which these lines all pass through certain points, and the numerals 1, 1 show the ventral segments which are then at their maximum displacement, the ventral segments 2, 2 and 3, 3 come to their maximum at different times, as already explained.

Again, suppose two sets of waves to be started in opposite directions, each set having the front $\perp$ to a side of the triangle, and the phases of both sets being the same along a perpendicular from an angle on the opposite side; we shall with a suitable wave-length have six sets of waves meeting each other in pairs, the directions of the pairs making with one another angles of 120°, and all the phases being the same at certain points.

Again, suppose waves to start simultaneously from each of the sides of the equilateral triangle; the waves will be reflected so as to produce other three sets of waves also with their fronts parallel to the sides, by having their direction of motion reversed. Here we again get the six sets of waves above considered, but in a different position. See fig. 6, in which this form of vibration is shown, without allowing for any change of phase at the reflections. The continuous lines show the coincidence of the maximum displacement in one direction due to the waves meeting; and the dotted lines show the same coincidence in the opposite direction. At the black spots we get the coincidence of all the maximum displacements in the same direction; so that these spots show the ventral segments. The wave-length is equal to the distance between the ventral segments $\times \frac{\sqrt{3}}{2}$. With the number of ventral segments shown in the figure the wave-length equals $\frac{1}{5}$ of the height of the triangle when no allowance is made for change of phase at the reflections. The form of vibration of the film is shown in fig. 4, which has been already explained.

We may obtain a similar set of waves in a rhombus having one of its angles $=60°$, if we suppose sets of waves to start

simultaneously from the four sides, and in each direction from the shorter diagonal.

If in such a rhombus a set of waves starts from the longer diagonal, we get the three sets; and if two sets of waves in opposite directions start from the longer diagonal, we get the six sets.

With a right-angled isosceles triangle we may start a set of waves from the hypotenuse, and so get two opposite sets of waves $\parallel$, and two opposite sets $\perp$ to the hypotenuse; and we may get four similar sets of waves in another position by starting waves simultaneously from the two sides of the triangle.

With a square we can get four sets of waves meeting two and two, the directions being $\perp$ to each other, by starting waves simultaneously from all the sides. In fig. 7 the continuous lines show the coincidence of the maximum displacement in one direction of two waves, and the dotted lines show a similar coincidence in the other direction. The black spots show the ventral segments which move together, and the small circles those which move in the opposite directions. The wave-length $=$ the shortest distance between the ventral segments $\times \sqrt{2}$; and with the number of ventral segments shown in the figure, the wave-length, not allowing for change of phase at the reflections, is $\frac{1}{3}$ of the side of the square.

A similar arrangement of waves in another position may be obtained from a square by starting two sets of waves in opposite directions from one of the diagonals.

To obtain four sets of waves meeting each other two and two, the angles between their directions being 2α, take a rectangle having its diagonals inclined at an angle 2α, and start two sets of waves from one of the diagonals; these will by reflection give two sets of waves with fronts parallel to the other diagonal.

With any rectangle two sets of waves meeting each other can be obtained by starting a set of waves from one side of the rectangle.

The case of two sets of waves meeting each other not in the same direction is impossible in a limited film; and I have not been able to discover any form of film which could maintain three sets of waves not making equal angles with one another,

or six sets meeting in pairs whose directions do not make equal angles with one another.

In the phoneidoscope we have a soap-film thrown into a state of vibration by a musical note. The effect is to send the matter of the film towards the ventral segments, and to make them the thickest part of the film. The consequence is that the colours of thin plates are seen less at the ventral segments than at other parts of the film; and we can recognize the ventral segments in this manner. This effect on the film may be illustrated by M. Decharmé's experiments on Chladni's plates*, in which he shows that if a thin layer of water instead of sand be spread over the plate, the water covers the ventral segments and the nodes are left bare.

The two following experiments may, I think, be explained by what has been said.

(1) A square film, $\frac{1}{12}$ of an inch in side, was thrown into vibration by a note having 92 vibrations in a second. The position of the ventral segments was that shown in figs. 5 and 7. Hence the vibration of the film was the result of four sets of waves starting simultaneously from the four sides of the film; and the wave-length was approximately $\frac{1}{3}$ of the side; and the wave-velocity was approximately $\frac{1}{3} \times \frac{1}{12} \times 92$ inches per second—that is, about 28 inches per second.

(2) An equilateral triangle, one inch in height, was thrown into vibration by a note having 152 vibrations in a second. The position of the ventral segments was that shown in figs. 4 and 6. Hence the vibration of the film may be the result of three sets of waves starting simultaneously from the three sides of the Δ, and giving by reflection three other sets moving in the opposite directions. And the wave-length would then be approximately $\frac{1}{5}$ of the height; and the wave-velocity was approximately $\frac{1}{5} \times 1 \times 152$ inches per second, or about $30\frac{1}{2}$ inches per second. Or the figure may be the result of *one* set of waves starting perpendicular to one of the sides of the triangle (see fig. 8). In this case the wave-length would be $\frac{3}{10}$ of the side of the triangle; and the wave-velocity would be $\frac{3}{10} \times \frac{2}{\sqrt{3}} \times 152$ inches per second, or about $52\frac{1}{2}$ inches per second. As this wave-velocity differs very much from the

* *Ann. de Chim. et de Phys.* sér. 5. vol. xvi. pp. 338–376.

wave-velocity derived from the experiment with the square film, we must reject this latter explanation.

The two experiments may be made to give the same wave-velocity by supposing a change of phase equal to half a period to take place at each reflection. In the first experiment the side of the square has to be taken equal to $\frac{5}{2}$ wave-lengths, and in the second the height of the triangle as equal to $\frac{9}{2}$ wave-lengths. The two expressions for the wave-velocity become $\frac{2}{5} \times \frac{11}{12} \times 92$, and $\frac{2}{9} \times 1 \times 152$, both of which expressions are equal to $33\frac{3}{4}$. Hence we are perhaps justified in inferring that the edges are stationary, and that the wave-velocity in the soap-film is nearly 34 inches in a second.

VII. *On a Means to determine the Pressure at the Surface of the Sun and Stars, and some Spectroscopic Remarks.* By EILHARD WIEDEMANN*.

IN a former paper I tried to show that we can calculate, at least approximately, the time elapsing between two encounters of the molecules of a gas by measuring the greatest difference in the length of path at which interference of two rays of light coming from the same source of light is possible. I may be allowed here to quote a passage out of that paper†:—

"Two rays of light can only interfere if they emanate from the same source of light, and if there is no sudden change of phase in the source of light during the time elapsing between the two instants at which the first and second ray leave the source. That difference in phase at which interference is still possible is therefore a measure of the time during which no sudden change in phase has taken place at the source of light. A luminous body sends out light coming from a great number of atoms or molecules. Each molecule will only vibrate regularly as long as it does not come within the sphere of action of another molecule; that is, it will only vibrate regularly in the time elapsing between two encounters. For the different molecules, the time elapsing between two encounters may, according to the molecular theory of gases, be either zero or

* Read June 12th.

† Wied. *Ann.* v. p. 503 (1878); Phil. Mag. [5] vii. pp. 79, 80.

infinitely large. But during a very short fixed time only a small number of molecules will have suffered encounters; only that small number will produce an even illumination of the field of view: the greater number will have vibrated regularly during the whole time; and for all of these interference is possible. The interference-bands in that case are sharp. If, however, the time which we consider to elapse between the emanation of the two interfering rays increases, a greater number of molecules will have suffered encounters, and the bands will therefore be less distinct. It follows that the higher the order of interference-bands, the more diffused and indistinct the bands will be. If the difference of phase corresponds to a difference in time greater than that necessary for the completion of the mean free path, the bands will rapidly disappear, as in that case the greater number of molecules have suffered encounters during the time considered."

Fizeau and Foucault, and, more recently, J. J. Müller and Mascart, have determined the greatest difference of path at which interference is possible. According to the last-mentioned observer, a difference in path of 50,000, and even 100,000 wave-lengths still produces appreciable interference-bands in sodium light. The time corresponding to this difference of phase is about $0{\cdot}5 \times 10^{-10}$ second. Calculating approximately the mean free path (as we only want to compare the orders of magnitude) for hydrogen at 0° and atmospheric pressure, we find for the time necessary to traverse this free path $1{\cdot}14 \times 10^{-10}$ second. The two numbers are sufficiently close to justify the assumption that the sodium atoms may vibrate during 50,000 oscillations without sudden change of phase. We see at the same time that the disappearance of Newton's bands need not be due to the widening of the lines *producing* them.

As the mean time elapsing between two encounters depends chiefly on the pressure, and far less on the temperature, we have a means of determining approximately the pressure of a gas by an examination of the light which it sends out.

The determination of the quantities relating to large difference of paths in the light sent out by the sun, its protuberances, and stars may give us important information on their physical constitution; and I should like to draw the attention of spectroscopists to this point, now that we may soon expect a renewal of the sunspot maximum.

In order to make the measurements, we need only decompose the light we want to examine by means of a spectroscope, separate a ray, which must be as homogeneous as possible, and count the number of Newton's rings visible between two adjustable pieces of glass. We might also determine the thickness of a plate of Iceland spar which still shows interference-bands if it is placed between two Nicol's prisms. The plate must be cut parallel to the axis; and particular attention must be paid to its homogeneousness.

The spectroscopic evidence hitherto has only related to the presence of a substance in the sun; and we only derive from it very general notions as to the physical state. The radiation of heat is, as Janssen has recently again had occasion to observe, a very complicated phenomenon. We must take account of all the different layers on the sun's surface; and the same difficulty besets the interpretation of the Fraunhofer lines. Their thickness and darkness is a function of at least three variables—temperature, pressure, and thickness of absorbing layer; only after having successfully investigated the separate effect of these three variables, will a more perfect interpretation of the phenomena be possible.

If electric phenomena are going on at the solar surface, the difficulty of the ordinary methods of investigation will be still further increased; for, as I have shown,

(1) Very often, if a spark traverses a mixture of gases, one gas only becomes luminous. This result has recently been confirmed by the photographs of H. W. Vogel.

(2) A gas may, by means of an electric discharge, be made luminous below 100° C.

I have concluded from my investigations that the electric discharge increases the oscillatory energy of a gas independently of its translatory motion; and I compared these phenomena to fluorescence.

Hasselberg has confirmed my results, and has drawn some conclusions from them concerning the aurora borealis, comets, &c. I did not in my first communication refer to these matters, as I hoped first to be able to make some experimental investigations in order to fix the relations existing between pressure, luminosity, and quantity of electricity; but I had thought of these evident applications.

The curious forms of prominences which rise and float freely over the solar surface, and which Lohse has tried to refer to chemical processes, may also be explained by electrical causes.

The two results which I have mentioned must render us very cautious before we apply results arrived at by means of the electric arc or vacuum-tubes to solar phenomena. I believe that we cannot at all employ them for the determination of temperature and pressure.

It is by no means necessary that we should have the same relative intensity of the lines in the spectra produced by the two different causes. The effect of the electric discharge is first of all to displace the æther spheres surrounding the molecules; and the vibrations which are caused are in the beginning independent of the translatory motion, which latter chiefly determines the temperature. By means of the encounters of the vibrating molecules the rotatory motion is changed into translatory motion; and then only is the temperature raised so high that the gas may become luminous owing to its heat. This may take place in the narrow parts of a Geissler tube.

If, on the other hand, we produce spectra by means of heating only without calling electricity to help, we first of all increase the translatory motion, which must be increased considerably; for the gas becomes luminous, and then the changes in the forces binding the atoms together to a molecule must affect the spectrum.

At any rate we must carefully investigate the effects of the electric discharge on the nature of spectra before we can draw any conclusions from spectra produced by electricity on questions relating to temperature and pressure. I intend to discuss this point in another paper.

VIII. *On a new Instrument for the Detection and Measurement of Inflammable Gas in Mines. By* E. H. LIVEING, *Assoc. R.S.M.**

[Plate II.]

THE instruments hitherto contrived for the detection of gas

* Read June 26th, 1880.

in mines may be divided under two heads:—1st, those which depend for their action on the *Physical* properties of the gaseous mixture; 2nd, those dependent on its *Chemical* or combustible properties.

Under the former head we have the instruments of Mr. Ansell and Prof. Forbes—the one depending on the diffusion of gases, and the other on the velocity of sound; while under the latter head we have the ordinary flame test, the instrument of M. Coquillion, and, lastly, my own instrument. Strong arguments can be urged to show the superiority of the latter class of tests over those of the former type; for it is the heating-value of the gaseous mixture, or its approach towards the explosive proportion, that we really want to know.

It is quite possible to alter the physical properties of a gaseous mixture to a very large extent while its combustible properties remain scarcely changed. Thus the presence of 6 per cent. of marsh-gas in atmospheric air is sufficient to render it explosive in an upward direction, and there may be present in the mixture amounts of CO_2 from 1 up to 15 per cent. without preventing the mixture being explosive.

Atmospheric air containing 8 per cent. $CH_4 + 8$ per cent. CO_2 is highly explosive; and yet both the instruments above mentioned as dependent on the physical properties would indicate such a mixture as perfectly harmless, indeed would fail to detect the presence of any combustible gas whatever.

There are other drawbacks to these instruments, but I will not occupy your time by detailing them. On turning to the other class we have, first, the flame test. This has, of course, been the practical every-day test employed in collieries for a century or more past, and was applied at a considerable risk before the introduction of the safety lamp, and since that time with comparative security.

The tail or cap observable upon a flame when brought into an atmosphere contaminated with combustible gas is nothing more than a region where the weak mixture of gas and air receives sufficient auxiliary heat to sustain its ignition-temperature, or, in other words, to burn. It is then a direct combustion test, and therefore on the right principle. There are, however, some drawbacks to it: there is a general feeling that it is not sufficiently sensitive; and many hold, further, that

there is a wide difference in the percentage of gas that may be present before the cap makes its appearance. Whether this is correct or not, I am not prepared to give a decided opinion ; but, from experiments made, I consider 2 per cent. of marsh-gas about the limit detectable with the ordinary Davy flame. A small and clean flame is an essential requisite in applying this test; any particles of ignited matter on the wick will readily produce a spurious cap when no gas exists.

Of the modes of rendering the cap more visible none is more efficient than turning the flame low, or hiding the luminous portion with some opaque object. Attempts have been made to increase the sensitiveness of this test by viewing the flame through blue glass, with a hope that, the yellow colour of the flame being subdued, the bluish cap would be the more apparent ; but this has failed practically to increase the sensitiveness of the test, and, I believe, for this reason—that as the percentage of gas is diminished the cap decreases more in size than in intensity; for it is only those molecules that come into very close proximity to the flame that obtain sufficient auxiliary heat to enable them to reach the ignition-temperature.

Of the instrument of M. Coquillion I have nothing to say; for though it may be a very convenient and neat apparatus for gas-analysis, the fact that it cannot be employed in the pit, and that samples of air have to be brought to it puts it entirely out of the present question.

I now come to my own instrument. The principle on which it is based is as follows :—A mixture of marsh-gas and air in which the marsh-gas forms less than 5 per cent. of the mixture is not explosive or capable of continuing its own combustion (at ordinary temperatures and pressures), simply because the heating-value of the CH_4 is insufficient to raise that large excess of atmospheric air to the necessary ignition-temperature. If, however, such a mixture is exposed to some sufficiently heated object, especially if that object is platinum, it will burn in its immediate contact and neighbourhood, and in so doing add materially to the temperature of the object, and the more so the larger the percentage of gas present. To apply this principle to

the detection and measurement of gas in the air of mines, I have contrived the following arrangement (shown in Plate II.) :— A and B are two spirals of very fine platinum wire, through which a current from a small magneto-electrical machine is made to circulate. Both wires being in the same circuit, and offering equal resistances and cooling-surfaces, become equally heated on turning the machine; one of these spirals is enclosed in a small tube having a glass end and containing pure air; the other is exposed within a small cylinder of wire gauze (also with a glass end) to whatever gaseous mixture is drawn within the instrument for examination.

So long as the air examined is free from combustible gas or matter, the two spirals glow with an equal brilliancy; but if the air contain any amount above $\frac{1}{400}$ of its volume of CH_4 the exposed spiral increases in brilliancy, and the more so the nearer the percentage approaches the explosive proportion; and this difference of brilliancy becomes a means of determining the percentage of gas present.

To measure the difference of brilliancy there is a simple form of photometer, consisting of a small wedge-shaped screen C, the two surfaces of which are covered with paper and illuminated, the one by the covered spiral, the other by the exposed one. These two surfaces are viewed through the small side tube G, which, together with the screen and outer tube K, can be moved towards the covered spiral until equal illumination of the two surfaces of the screen is attained in the particular gaseous mixture under examination. An empirical scale is then read off, which gives at once the percentage of gas present.

The following table shows the difference of brilliancy due to stated percentages of CH_4; but the amount much depends on the size of the wires, proportion of the spiral, and other conditions, and can only be determined by actual trial in known mixtures. It is a point worthy of note that the rate at which the machine is turned does not seem to have any sensible effect on the ratio of brilliancy of the spirals so long as the exposed one is heated to the igniting-point of CH_4.

Table of observed Illuminating-power of the Spirals in different percentages of gas.

Per cent. CH_4.	Relative illuminating-power of spirals.	
	Covered spiral.	Exposed spiral.
Pure air.	1	1
¼ per cent. CH_4.	1	1·24
½ " "	1	1·65
1 " "	1	2·78
2 " "	1	5·1
3 " "	1	22
4 " "	1	64

When moderately large quantities of gas are present, such as two or three per cent., the difference in colour of the light emitted by the two wires becomes so striking as to cause some difficulty in judging of the exact position of equal illumination of the screen, a difficulty of course felt more or less with all photometers when examining lights of different tint. This difficulty is avoided (so far as the object of this instrument is concerned) by covering one half (Y, fig. 2) of that side of the screen that is illuminated by the exposed spiral with yellowish red paper, the other half, X, remaining white; and for quantities below 2 per cent. the white surface X is compared with the white surface Z; but above that amount the yellow surface Y is employed, its tint neutralizing the very white light of the exposed wire and rendering the comparison easy.

I must now say a few words relative to the practical use of the instrument.

In the modern systems of colliery-ventilation the main current of air that descends the downcast shaft is soon subdivided into several separate currents (or "splits," as they are called). These, passing along the principal travelling-ways, are conveyed to the different districts where the men are working, and, after ventilating a certain number of working-places and becoming more or less contaminated with the combustible gas that is there evolved from the freshly exposed surface of the coal, are carried by air-ways known as "returns" to the upcast shaft, often skirting in this part of their course large areas of broken ground (or goaf) where the coal has been entirely removed.

Now it is for the examination of the air passing in these different returns that I consider this instrument specially adapted, as it at once enables the manager of a colliery to determine the quantity of gas evolved in the different working-districts, and thus regulate the proportion of his various currents of air so as to make the best use of the given ventilating-power that he has at his disposal, and not have some of his air-ways considerably contaminated with gas whilst others are practically free (a condition I have frequently found to be present). Again, if a regular account of the observed percentages of gas be kept, the quantity of air passing and the barometric pressure being also noted, it will soon become evident within what limits it usually varies; and should at any time an abnormal increase be observable, it will be desirable to follow up the return until the source is ascertained and precautions, if necessary, taken.

The instrument thus employed, and the results carefully considered, cannot fail to be of assistance in laying out and regulating the ventilation of extensive collieries. There is, however, another and perhaps more urgent requirement which I hope a slight modification of this instrument may also be able to meet.

There exist many gassy collieries where safety lamps are alone employed, and yet where the hardness of the coal necessitates the use of powder in order that it may be worked at a profit. Now in such workings, before each shot is fired, it is necessary for the sake of safety (and enforced by the Coal-Mines Regulation Act) that the working-place and its neighbourhood should be carefully tested for gas. This is of course done at present by aid of the lamp, and, it is to be feared, too often without that degree of care and deliberation by which alone the test can be considered efficacious. Be this as it may, all are agreed that some more striking and definite test would be of great value for this purpose, if it could be practically introduced.

For this object I propose a slight modification of the instrument described, in which the screen is fixed at some definite percentage, such as 2 per cent., the one side being marked G, the other A; the directions for use being that so long as the

side A of the screen is the brightest the shot may be fired, but if G become equal to or brighter than A the shot should not be fired.

For this purpose the instrument and machine would be best made in one piece, and would have to be constructed as *strongly*, *simply*, and *cheaply* as possible, with an easy means of replacing the platinum spirals, should they be melted through carelessness.

I may add that the chief difficulty about the introduction of the instrument is to find a sufficiently compact and portable source of the necessary electrical current that can be made at a moderate price.

There is yet one other possible application of the instrument —namely, for the examination of the heating-value of the waste gases from blast-furnaces, where it is of importance to regulate the conditions of burning so as to reduce the carbonic oxide in the waste gases to a minimum. Of course the sample of waste gas would have to be mixed with some definite quantity of air, to supply oxygen, before examination.

Explanation of Plate II.

R and S are two strong brass plates that form the terminals of the electrical machine and at the same time act as supports for the instrument.

D and E are wooden plugs with copper wires carrying the platinum spirals A and B; these are arranged so as to be easily replaced in case of accidental melting or damage. For this purpose the thumb-screws L and M are removed, and the whole instrument turned half round upon the screws O and P; the plugs being in this position released, can be replaced by fresh ones, two small springs making the requisite contacts without trouble. The path of the current is shown by arrows.

There are two small entrance-tubes, through which a breath is taken to fill the instrument with the air to be examined. These are not shown in the figure, not being in the plane of section.

Q is the magneto-electrical machine, the dimensions of which are $8 \times 5 \times 1\frac{3}{4}$ inches.

IX. *On the Behaviour of Liquids and Gases near their Critical Temperatures.* *By* J. W. Clark.

[Plate III.]

In 1822 Baron Caignard de la Tour * showed that the effects of heat on a liquid enclosed in a vessel adapted to the purpose, was to convert it into vapour at a volume a little more than twice that which it originally possessed. Within a certain limit he found the temperature at which this change occurred was independent of the ratio existing between the volume of the liquid and that of the tube, but above that limit the conversion was first observed at a higher temperature. Brünner †, after an extensive discussion of previous memoirs, states the results of his observations upon the decrease which heat produced in the height to which water, ether, and olive oil rose in capillary tubes. He experimented at temperatures below 100° C. In 1857, from Brünner's expression for this decrease in the case of ether, Wolf ‡ calculated the temperature at which the level of the liquid in the capillary tube should coincide with that outside it; and in the attempt to test his conclusion experimentally he found that it sank below the liquid in the external tube. He stated that the surfaces became convex, and that the depression was the result of true capillary action, as in the case of mercury. Drion § then took up the subject, and ultimately came to the conclusion that the depression of the liquid in the capillary tube just before its vaporization was the result of a less rapid expansion produced by a difference of temperature too small to detect with a thermometer. He described the surface of the liquid at the moment of disappearance as perfectly plane, and the apparent convexity as the result of refraction.

Mendelejeff ‖ appears to have misunderstood Wolf; for in 1870 he writes :—" Wolf hat im Jahr 1858 (*Ann. de Chim. et de*

* *Ann. de Chim. et de Phys.* t. xxi. p. 127 and p. 178; *ibid.* t. xxii. p. 411.

† Pogg. *Ann. der Phys. u. der Chemie,* Bd. lxx. S. 481.

‡ *Ann. de Chem. et de Phys.* t. xlix. p. 272.

§ *Ann. de Chim. et de Phys.* 1859, p. 221; and *Comptes Rendus,* t. xlviii 1859.

‖ Pogg. *Ann.* 1870, Bd. 141, S. 621.

Phys., t. 49, p. 259) gezeigt dass bei der Temperatur bei welcher Aether in zugeschmolzenen Röhren ganz in Dampf verwandelt wird der Meniscus verschwindet und dass Niveau des Aethers in der Capillarröhre und in der weiten Röhre gleich ist. Die Beobachtung wurde von Drion (*ibid.* 1859, t. 56, p. 221) bestätigt und erweitert." The researches of Dr. Andrews * are too well known to need special reference here; and the discussion of a paper by Dr. Ramsay †, on the "Critical State of Gases," will be best left until the completion of the work in part described in this paper. Messrs. Hannay and Hogarth ‡ have recently shown that under certain conditions solids may be dissolved in gases. Unfortunately, Mr. Hannay's paper, "On the Cohesion Limit," recently communicated to the Royal Society, is not yet printed; so that reference to it must be left until a future occasion.

In December 1878 I read a short preliminary note before the Society, "On the Surface Tension of Liquified Gases," in which the results of some measurements on sulphurous anhydride at low temperatures were given §. Last year, at the Society's meeting at the Royal Indian Engineering College in June, a curve showing the height at which sulphurous anhydride stands in a tube at temperatures between −17° C. and the critical temperature was shown, and the depression of the liquid in a capillary tube, with some unsuccessful attempts to determine the cause of it, were described. As this is the last meeting of the session, I beg leave to lay the result of the inquiry before the Society.

Two methods of heating will be noticed in this paper. The first consists merely in heating a large test-tube of oil over a rose gas-burner, the experimental tube being suspended vertically in its axis a little above the bottom by a fine platinum wire. Numerous experiments have been tried with U-tubes of which the branches were of *equal diameter and glass-thickness*;

* Phil. Trans. 1869, p. 575; *ibid.* 1876, p. 421.

† Proc. Roy. Soc. vol. xxx. p. 323.

‡ *Ibid.* xxix. p. 324; *ibid.* xxx. pp. 178 and 188.

§ I have since found that the value then given is considerably too low, no correction for the diameter of the external tube having been made. To Professor Quincke I am greatly indebted for having directed my attention to the previously reasoned observations of Wolf and Drion here quoted, and for having kindly supplied me with most of the above references.

and the results show that the heating, although rapid, is nearly, if not quite, uniform. It is worth notice, however, that at temperatures very near the critical, the density of liquid and vapour are so nearly equal that when the heating is made unequal, the entire disappearance of the liquid from one branch of the tube does not affect its level in the other. The second method of heating is more complex, but has answered the purpose sufficiently well to deserve brief description. In the middle of a four-litre beaker of oil* is fixed a round-bottomed thin flask of 1·5 litre capacity. The neck, which is about 3 centims. in diameter, is thickly wrapped round with cotton wool and calico, and is long enough to rise 7 centims. above the perforated cover which closes the beaker. The beaker rests on wire gauze carried by a large iron ring, and is surrounded by a tin cylinder 8 centims. wider than the beaker, furnished with two small glass windows on opposite sides. The neck of the flask passes through the cover of this outer case, and is closed air-tight by an indiarubber cork, through which the bulb of a long-stemmed thermometer is inserted. Just below the cork is fused into the neck a narrow glass tube connected with a slightly modified Bunsen regulator (fig. 1, Plate III.) by a short piece of composition pipe (a) 2 millims. in diameter†. The other end communicates through a short glass tube (b) with the air-mercury bulb of the usual regulator. A small glass stop-cock (c) serves to place the flask and regulator-bulb in communication with the air. A large rose burner protected from air-currents heats the beaker of oil.

When the apparatus is in use the experimental tube is suspended in the middle of the flask by a thin wire, the glass stop-cock (c) of the regulator opened, and a small gas-flame lit beneath the beaker. When ether is used in the experiment, in from eight to ten hours the temperature rises to within

* The best salad oil heated out of contact with such metals as copper and iron will bear repeated heating to 200° C. It ultimately becomes thicker and denser, and exhibits a green fluorescence. By long exposure to the action of air and light, particularly in thin sheets, it regains its transparency and again becomes fit for use. It should be noticed that some sorts of glass are violently attacked by hot oil.

† These regulators may be obtained from Mr. Bel, 34 Maiden Lane, Strand.

three or four degrees of the critical; the glass stop-cock (c) is then closed, and the pressure of the expanding air in the flask begins to raise the mercury until it meets the platinum tube (e) of the regulator, which is previously screwed down till within about 3 millims. of the surface of the mercury. Some two or three hours later the critical temperature is reached. The light from a paraffin lamp passes through a cylindrical lens, and enters through one of the glass windows in the case enclosing the apparatus, thus rendering the experimental tube visible through the other window. The observations are made with a cathetometer-telescope.

Early in this inquiry it was found that so many circumstances influenced the depth to which the liquid was depressed in the capillary tube, that two different tubes could be satisfactorily compared only when enclosed in the same external tube; and if the effects of slow and rapid heating are to be compared, the tubes should be of the same thickness of glass. Small springs of thin platinum (S, fig. 2) fix the capillary tubes vertically in the axis of the external tubes, which latter have been employed of various diameters between 2 and 20 millims. When the slow heating-apparatus is used, the experimental tube (enclosing the capillary tube) is fixed in the axis of a tube of considerably greater diameter, which is then exhausted of air and hermetically sealed (see fig. 3). Ether distilled from calcic chloride has been the liquid most frequently employed; but, so far as I am aware, all the results about to be described are also obtained with alcohol, sulphurous anhydride, and carbonic disulphide. Water attacks glass so rapidly that it is difficult to ascertain what is taking place in the tube; but it probably forms no exception to the liquids mentioned.

After the introduction of the capillary, the external tube is filled with liquid, and the air removed as completely as possible by repeated boiling, and the volume of the liquid is reduced to a convenient extent. When such a tube contains ether and is heated, the liquid sinks in the capillary tube and rises in the outer, the expansion becoming more and more rapid as the critical temperature is approached. About 2° C. below this temperature the meniscus in the capillary tube stands at the same level with that in the external tube. Both surfaces are then distinctly concave. In the case of alcohol

the temperature at which this is observed is *probably* a little more, and for sulphurous anhydride a little less, than 2° C. below the critical temperature. The exact temperature is affected by a number of circumstances : thus it is lower with a wide capillary tube than a narrow one, and for the same capillary tube it is also lower when this tube is deeply immersed in the liquid. Continuing the heating, the meniscus (*c*) in the capillary is seen below that (C) in the external tube (fig. 4). It then gradually loses its concavity, becomes successively plane and less defined, and frequently presents a more or less convex appearance before finally vanishing. Meanwhile the liquid in the external tube expands and undergoes the same changes, the defined surface appearing more or less convex, and then becoming black and ill-defined. It first ceases to be visible in the capillary tube. The curve fig. 5, Plate III., shows the changes in the height of liquid sulphurous anhydride in a capillary tube of 0·4358 millim. diameter in an external tube 3·2 millims. wide. The abscissæ are in degrees centigrade, the ordinates in millimetres. The measurements near the critical temperature could only be made with difficulty, and are consequently less reliable than those made at a lower temperature. Measurements of the *depression* of the liquid, or even of the diameter of the capillary tube in which the depression is observed, possess but little value, other conditions (*e. g.* rate of heating, &c.), incapable of exact expression, exercising too great an influence upon it. When the tube is rapidly cooled a local cloud of very fine particles suddenly makes its appearance, in the middle of which the liquid first appears as a fine horizontal line (fig. 4). In the capillary tube this cloud extends to a considerably greater distance, both upwards and downwards, than in the external tube. Probably this is connected with a surface-action, to be described subsequently. The mean position of the cloud is nearly that at which the tube-contents disappeared on heating. The formation of this cloud reminds one forcibly of the sudden crystallization of a supersaturated solution. The volume which the liquid then first occcupies is less than that which is possessed before its disappearance, and is considerably less if the black ill-defined surface visible on heating be taken into consideration. A persistent tendency on the part of the ether and sulphurous

anhydride gases to condense at a lower temperature (particularly when somewhat rapidly cooled) than that at which they were formed is very noticeable *. Rapidly cooled (especially in narrow external tubes) the liquid, when it first becomes visible on condensation, is either level in the capillary and external tubes, or higher in those tubes in which the depression on rapid heating has been the greatest. When slowly cooled, or even when somewhat rapidly cooled in an external tube of large diameter, the liquid condenses in greater quantity in the external tube than in the capillary tube, and hence the liquid has to rise in the capillary tube before it reaches the level of the liquid outside. This is easily seen to be the case from the shape of the condensing cloud in an external tube 20 millims. in diameter. It will be subsequently seen that the resistance which the narrow tube offers to the flow of the liquid through it, is at least very closely connected with the cause of the depression †.

When the tubes are very slowly heated in the apparatus already described, the above phenomena are considerably modified. The volume occupied by the same liquid before its vaporization is then seen to be far greater than that occupied by it when rapidly heated. To illustrate this the volume of ether in a certain tube at the ordinary temperature was roughly 25. Rapidly heated, the volume at which the *liquid* disappeared was between 35 and 40; and at a volume of about 45 the last traces of black cloud had gone and the contents of the tube appeared perfectly homogeneous. Slowly heated when the liquid had expanded to a volume of 52, the meniscus was slightly, but perceptibly, concave, and disturbed by a rising bubble of gas. The enclosed capillary tube was not long; and the liquid expanded until it reached the top, and then poured into the tube and filled up the existing depression. Indications that a higher temperature is required for this in-

* Although it is premature to draw any definite conclusion from this, it may be of interest to recall the researches of Magnus (Pogg. *Ann.* xxxviii. 592), and Regnault (*Mémoires de l'Académie* 1862, xxvi. 715–749, 335–664) on the vapour-pressures of liquid mixtures, which seem to bear a possible relation to it. In this connexion see also Cailletet's paper (Phil. Mag. March 1880, p. 235).

† For this idea I am indebted to a suggestion made to me by my friend Mr. Eagles.

creased expansion of the liquid have been observed; but whether the temperature at which the liquid disappears is the same for rapid and slow heating has not yet been satisfactorily determined. Continuing to heat, the liquid expands, and the surface is reduced to a thin and often waving line, obliterated by further expansion, or lost amongst the frequent striæ. Under these circumstances the volumes of disappearance and reappearance are nearly equal, and the corresponding temperatures sometimes differ by less than 0°·1 C. A very slight sudden rise of temperature, when the liquid has expanded beyond the volume at which it disappears under rapid heating, suffices to replace the defined surface by the black ill-defined one before described; but, apparently, when this expansion has proceeded too far, the surface then becomes broad and ill-defined, but not black. It then passes from the liquid through this ill-defined state without expansion; and the liquid in a rather wide capillary tube is then seen to be level with that outside it. Slowly cooled, the tube frequently becomes uniformly filled with striæ, and the light transmitted through the experimental tube by the paraffin lamp before described is observed to become gradually redder and redder. This is succeeded by the formation of a whitish incipient cloud, which finally precipitates in visible particles, often throughout the whole tube. The liquid contracts from the first moment of its condensation until it regains its original volume. Slow and regular cooling seems more difficult to attain than the corresponding conditions on heating.

When the external tube contains rather less ether than the above—that is, when about one third filled, and rapidly heated, the liquid expands and passes into gas in the usual way. Very slowly heated the liquid also expands; but after reaching a certain maximum volume, it very gradually diminishes and evaporates away. If the volume of liquid at its point of maximum expansion happens to be such that the liquid in the external and capillary tubes are almost level, the meniscus in the capillary tube shows a slight tendency to increase its height above the surface when the contraction commences. A slight sudden rise of temperature produces a result similar to that described in the last case. On cooling, the liquid in such a tube undergoes a momentary and almost inappreciable increase of volume just after condensation, and

then contracts to its initial volume as in the previously described tubes. If a tube contains still less ether than the last mentioned, the liquid undergoes a more marked increase of volume when it first condenses; and this continues longer before the normal contraction sets in.

Very near the critical temperature the density of liquid and gas are almost equal; and hence it is that the meniscus in the capillary tube may remain depressed for as much as an hour and a half, although very slowly following the upward motion of that in the external tube. The meniscus in the capillary tube usually fades away a little before the surface ceases to be visible in the external tube. This may be due to its being under a slightly lower pressure.

The meniscus in a wide capillary tube is observed below the surface of the liquid in the external tube before that in a capillary of small diameter ; and rapidly heated, the depression usually remains greater in the wide tube until the surface ceases to be visible. This disappearance frequently takes place first in the small, then in the larger capillary, and lastly in the external tube ; when the heating is sufficiently slow, the depression becomes greatest in the tube of small diameter. Slow or rapid heating and slow cooling alike show the depression is greater in a tube roughened by hydrofluoric acid than it is in a smooth one of the same size. When the heating is rapid, the depression in a capillary tube the immersed half of which has been previously heated and drawn out until the diameter is very small is less than it is in the corresponding tube of uniform diameter throughout. Slow heating reverses this result, as shown in fig. 3. By the mere contraction of the extreme end of the immersed part of a capillary tube the depression is almost uninfluenced. The results above described are not invariable (and some points connected with them are still the subject of inquiry); but under conditions which are apparently the most free from error they become sufficiently so to justify their being regarded as normal.

It seems probable that the effects of unequal and irregular heating are completely removed from the above experiments; for the liquid is equally depressed in two capillary tubes of equal diameter, one made of very thick, and the other of very

thin glass. Rapidly heated, the depression reaches a maximum in the thick glass tube. Of two similar capillary tubes the most deeply immersed always shows the deepest depression; but the length of the tube above the liquid is apparently without influence. This led to the result described to the Society in June 1879, viz. that when the capillary tube does not dip more than a millimetre or two below the surface the liquid disappears at the same level in the capillary and external tubes.

It has been previously stated that the liquid sometimes appears convex near the critical temperature. With ether in a tube 20 millims. in diameter, this apparent convexity is so well marked that I had at first much difficulty in satisfying myself that such was not the case. By means of a lamp, cylindrical lens, and slit, a bright line of light is easily thrown into the liquid in such a tube heated in a large beaker of oil. The changes in the form of the reflected image were then observed ; and from this it was inferred that when the liquid surface appears convex and well-defined, as seen through the horizontal telescope, it is slightly but unmistakably concave, and remains so until it loses the power of reflecting when it is plane. This apparent convexity is caused by the raising of the far edge of the liquid by refraction; or perhaps it may resemble mirage, as I have distinctly seen the *surface* of the liquid in the horizontal telescope. The surface subsequently becomes black and ill-defined, and, as Professor McLeod has pointed out to me, closely resembles the surface of alcohol and water in a test-tube into which they have been carefully poured so as not to mix. The band of light which leaves the cylindrical lens, and passes through the vertical tube of ether, gives rise to some remarkable refraction figures at the surface of the liquid. By substituting the test-tube of alcohol and water for the tube of ether, corresponding figures are obtained—heat in one case, and diffusion in the other, causing these figures to pass through the same changes. It appears possible that the black surface may be due to the mixing of the liquid ether with its vapour when they are of nearly equal density. This view receives some support from the fact that, just before the defined surface of the liquid is lost, the convection-

currents become so rapid and violent as to disturb and apparently almost break through it into the vapour above.

The above conclusion as to the form of the surface receives confirmation from some experiments with an external tube enclosing two glass plates 0·15 millim. thick, and 11 millims. wide, and separated from each other by a short distance. Not the least rounding of the surface of the liquid could be detected either between or around the plates. That no depression of the liquid was observed between the plates, corresponding to that in the capillary tubes, may be explained by the very small expansion of the liquid in this tube at that point at which the depression usually takes place. Under the same conditions the depression is absent also from a capillary tube. The expansion, as already stated, is dependent upon the amount of liquid in the tube and upon the rate at which it is heated. On cooling, just as the cloud appeared, and before the liquid line had made its appearance in it, an interesting action was observed taking place on the surfaces of these glass plates; for, extending downwards into the cloud and considerably aoove its upper limit, a liquid film could be seen running over their surfaces. This probably affords evidence of a surface-action preceptibly influencing the position at which the condensation of the liquid takes place.

It remains only to briefly notice a class of facts to which reference has not yet been made, but which includes certain conditions capable of modifying some of the results described in a part of this paper. It has been very frequently observed that when a tube is heated for the first time, the depression is smaller than it is when the tube is again heated within a short time of the first experimnnt. In a few capillary tubes the liquid is seen to disappear at the same level as the liquid outside them; but reheating shortly after the liquid has condensed, the usual depression is observed. When such a tube has been left a sufficient length of time in contact with the liquid on heating, the same result is obtained as at first; this may be repeated indefinitely. For two tubes this time has been determined, and in both cases found to be about 20 hours; a shorter period merely sufficed to *diminish* the depression. Probably closely connected with this action is the gradual

decrease Quincke * has observed that time produces in the form of a bubble of gas in a liquid, and of a drop of mercury. To this class of facts are also nearly related the decrease in the intensity of Quincke's † electrical diaphragm-currents, and that which ‡ I have shown to take place in the electromotive force produced when water is forced through capillary tubes. Elster § has recently extended the observation to a similar variation in the electromotive force set up by liquids flowing over the surfaces of solids. Dorn ‖ has investigated at some length the cause of this action in the case of tubes, and has shown that it is capable of modification in various ways, some of which appear capable of exercising a corresponding control over the above-described depression of a liquid in a capillary tube at a temperature near the critical.

It is proposed to continue the still incomplete portions of this inquiry in a paper to the Society next session.

In conclusion I beg leave to express my thanks to Professor McLeod, not only for having advised me to extend my observations to higher temperatures than those at first employed, but also for the willingness he has always shown to aid me with valuable suggestions.

Summary of Contents.

1. When a tube enclosing a capillary tube dipping into alcohol, ether, or sulphurous anhydride is heated, the liquid sinks in the capillary, and rises by expansion in the outer tube. Between 2° and 3° C. below the critical temperatures of these liquids both surfaces become level; and on continuing to heat, the concave meniscus in the capillary tube is seen below that in the external tube. The extent of this depression depends on the diameter &c. of the capillary tube, and on the nature of its internal surface. When the end of a capillary tube dips very slightly below the surface of the liquid, it is level in the capillary and external tubes at the disappearance of the liquid.

* Pogg. *Ann.* Bd. clx. S. 576. † *Ibid.* Bd. cx. S. 56.

‡ *Ibid.* 1877, S. 345.

§ *Inaugural-Dissertation über die in freien Wasserstrahlen auftretenden electromotorischen Kräfte.* Leipzig, 1879.

‖ Wiedemann's *Ann.* Bd. ix. 1880, S. 523. Compare also Helmholtz, Wied. *Ann.* vii. p. 337 (1879).

2. In some capillary tubes the liquid is not depressed, but disappears at the level of the liquid in which they are immersed on first heating. Once heated, long contact between liquid and tube is necessary to prevent the formation of the depression on again heating. For two tubes which were examined, this time was in each case about 20 hours; a shorter period merely sufficed to *diminish* the depresion. The depression is the result of an action between the liquid and the inner glass surface of the capillary tube.

3. Indications that surfaces exercise a slight action in determining the position at which the liquid condenses in the external tube have been observed.

4. By reflecting a bright line of light from the apparently convex and well-defined surface of ether in a tube of 20 millims. diameter at a temperature near the critical, it may be inferred to remain concave until it loses the power of reflecting when it is plane. The apparent convexity is the result of refraction, or, perhaps, of an action resembling mirage.

5. The black ill-defined band which immediately succeeds the disappearance of the liquid surface is the result of too rapid heating, and possibly due to the mixing of liquid and vapour when they are of nearly equal density. When very slowly heated, as described, the defined concave surface is gradually obliterated, and is last seen as a fine and often waving line. Under this condition also the volume of the liquid at its disappearance is greater than when it is rapidly heated. When the liquid is vaporized by rapid heating, it has a higher temperature and larger volume at the time of disappearance than it has when first condensed by cooling; slowly heated and cooled, these volumes and temperatures are more nearly the same.

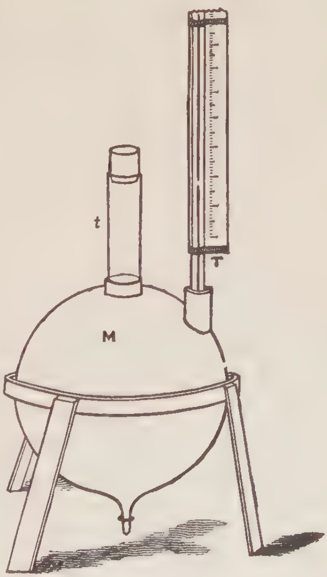
Royal Indian Engineering College,
June 1880.

X. *Note on a Modification of* Bunsen's *Calorimeter.* *By* W. W. Gee *and* W. Stroud*.

Professor Stewart described to the Manchester Literary

* Read June 26.

and Philosophical Society, on March 4, 1879, a calorimeter devised with the purpose of obtaining the specific heat of a substance of small quantity with much readiness. It consists of a combination of part of Bunsen's arrangement with that of Favre and Silbermann. In it advantage is taken of the method employed by Bunsen—namely, that of dropping a small body, whose temperature does not differ much from that of the atmosphere, into ice-cold water contained in a small tube. In Bunsen's instrument the heat so given up by the body experimented with is measured by the change of volume produced by the melting of ice surrounding the tube. There being certain practical difficulties in the use of this method, it was thought that if the tube were surrounded by a large mass of mercury, forming the bulb of a delicate thermometer, after the manner of Favre and Silbermann, then the direct expansion of the mercury would indicate the amount of heat brought into the tube by the body, this mercury being in its turn surrounded by melting ice not in contact with it. The instrument figured was made by Casella. T is the graduated tube of a glass thermometer whose large bulb is encased in the copper chamber M, there being only a small air space between the two. In the centre of this large bulb a test-tube *t*, provided with a cork, is tightly inserted, so that its lower portion is completely surrounded by the mercury in the bulb.



Prof. Stewart intrusted the calorimeter to us, in order that we might determine its working conditions and ascertain how far it was reliable.

The following was the way in which the apparatus was tested. The test-tube was first filled with water to the level of the mercury inside the

bulb. The instrument being placed in its wooden case, the latter was then filled with pounded ice and then allowed to cool down until the column of mercury in the stem of the thermometer T was apparently stationary between two observations taken at an interval of about fifteen minutes.

After preliminary trials with various quantities of mercury dropped into the test-tube, it was found absolutely necessary to make some allowance for the loss of heat from the surface of the mercurial thermometer by radiation and convection; for although the rise produced by dropping in was almost immediate, yet during a comparatively long period of apparent maximum that followed, the heat received must have been equivalent to the heat lost by radiation and convection. (This prolonged stationary period, it was always observed, was followed by a comparatively rapid fall.) A curve of cooling, extending from 0°·3 to 0°·0 C., was accordingly obtained, with times as abscissæ and temperatures as ordinates. By aid of this curve the loss of heat was allowed for by a simple method of compensation.

Brass and mercury were selected for comparison. The results, after applying the correction above indicated, for 2 and 6 grams of mercury and 2 grams of brass are given below:—

2	grams	mercury at 15°·8 C.	Corrected rise	15·6
2	„	brass at 15°·9 C.	„ „	46·0
6	„	mercury at 15°·25 C.	„ „	48·5

An inspection of these results, knowing that the specific heat of brass is about three times that of mercury, will show that the principle of the instrument is experimentally verified. The bore of the thermometer-tube being, unfortunately, large, necessitated a small rise; greater accuracy hence could not be expected. It is hoped shortly, however, to make further experiments with a new instrument having a much smaller bore, and with other needed improvements that could not in a first instrument be foreseen as wanting.

XI. *On Magneto-Electric Induction.*—Part II. *Conductivity of Liquids.* *By* Frederick Guthrie *and* C. V. Boys, *Assoc. R. Sch. Mines**.

[Plate IV.]

In a previous communication † we showed experimentally that, other things being equal, a conductor in a moving magnetic field is urged to move by a force which varies as the product of the conductivity into the relative speed; so that by observing the torsion of a wire supporting successively a variety of conductors of the same form and size in a revolving magnetic field, a measure of their relative conductivity may be obtained. In the case of most metals, this method of determining conductivity cannot be compared, at least for convenience, with the usual one with the bridge, galvanometer, &c.; but with less-perfect conductors, with such as cannot be drawn into wire, and especially with electrolytes, our method seemed very promising; for, whatever be the actual course of the electrical currents induced in the liquid, they are closed, no electrodes are present, there is no electrolysis, and there is no polarization.

The preliminary experiments, performed with the apparatus described in our last paper, clearly showed that with suitable means a measurable and even a large effect might be produced. Accordingly the experiments were continued with the very powerful and delicate apparatus shown in Plate VIII. One litre of liquid is put into the vessel A, which consists of a glass shade, the open end of which has been roughened with emery, and a strip of strong brown paper attached to it with paste. Two vertical strips of ebonite are fastened to the paper ring with shellac, and bound round with cotton, and the whole well varnished with shellac varnish. These strips are fastened by ivory pins to the boxwood beam B; and to the centre of this beam the torsion wire w is made fast by being wound three times round the little reel r and the end secured by a needle-point. The torsion wire passes over a zinc wheel

* Read on June 26th.

† Proceedings of the Physical Society of London, vol. iii. part iii. p. 127; Phil. Mag. December 1879, vol. viii. p. 449.

about 13 feet above the beam B, and down again to the same level, where it passes round the zinc screw z; so that by turning this screw the vessel A can always be brought to the same level, whatever be the specific gravity of the liquid within it. To the top of this vessel are attached two glass arms, one on each side, one carrying a photographically-reduced scale, s, on glass, and the other a counterpoise. The vessel A is entirely enclosed in a screen shown dark in the Plate. The thick portions of the screen are of wood; but the thin cylindrical part immediately round A is of thin ebonite, and the hemispherical part below A is of glass. This screen prevents the whirlwind of air caused by the revolving magnets from interfering with the motion of the vessel A. It also supports a microscope, H, provided with cross-wires, which is used to read the position of the scale S. The magnet consists of 24 semicircular bars put together as shown, so as to form a ring, the similar poles of the different bars being kept from touching one another by the fourteen pieces of iron shown dark in the plan. The magnet is $7\frac{1}{2}$ in. in external diameter, and weighs about 18 lbs. The magnetic field within it is remarkably uniform; iron-filings on glass take the form shown by the streak lines in the plan. The magnets are attached by four brass plates to the gun-metal wheel, G, which screws against an accurately turned shoulder on the vertical steel shaft, I. This shaft runs in cylindrical split gun-metal bearings; but its weight is taken by a slightly convex hardened steel round-headed pin let into the shaft and resting on a plate of quartz. The framework which carries the bearings consists of five triangles of cast iron screwed together in the form of a distorted octahedron, a form which gives great rigidity; it is omitted in the Plate to avoid complication; but its general outline is shown by dotted lines s. The lower triangle of the framework is screwed to a thick slab of mahogany, which is clamped to the stone mantlepiece of the room; and one hundredweight of iron stands on the wood to increase its inertia and make the whole thing more steady. Near the top of the shaft I a fine screw is cut, which gears with the worm-wheel m of 100 teeth. This wheel has also cut on its edge a screw of the same pitch as that on the shaft I; so that its edge is divided into a series of square pyramids, and it gears

as a screw into the worm-wheel n of 100 teeth : n therefore turns round once for 100 turns of m or for 10,000 turns of I. At every turn m makes an electric contact and sounds a bell; n has its face divided and numbered. Therefore by observing occasionally the exact instant at which the bell sounds, and then the reading on n, a continuous record of the speed can be kept, at the same time that s is being observed with the microscope. A pulley-wheel on I is connected by a catgut band with a small steam-engine driven by steam from the boiler which supplies the building. The steam-pressure varies so much that the resulting changes in speed in the engine have caused the greatest inconvenience, sometimes a whole day being lost. Some form of absolute governor will have to be devised in order to avoid great loss of time.

On the lower side of the beam B is a small glass hook, from which may be hung a disk of brass D, the use of which will be explained below. The dimensions of most of the parts of the apparatus may be found from the Plate, which is drawn to the scale of almost exactly $\frac{1}{4}$. The torsion-thread is of hard drawn steel wire about 13 feet long and 0·007 in. in diameter. The space in which the cylindrical part of the screen is placed between the poles of the magnet and the outside of the cell is only about $\frac{3}{16}$ in. wide, so as to get the greatest possible effect from the magnets. When the machine was put together the magnets were found to be slightly out of balance; so a plate of brass full of tapped holes was fixed to the light side, insufficient in weight to restore the balance, and then small screws of brass were screwed into the holes until the balance was perfect. By this means the magnets were made to run so smoothly that at 3000 turns a minute scarcely any tremor could be felt; but always on passing a certain critical speed (about 1300 turns a minute), at which the period of vibration of the whole machine and mantlepiece was probably equal to the time of rotation of the magnets, the machine set up a vibration, gentle at first, but increasing with the speed, which seemed at first as if high speeds could never be attained. But it was found that if, as soon as this effect took place, one of the screws of the upper bearing was loosened the oscillation ceased suddenly, and then the screw could be tightened again, and the speed increased to any extent without the

slightest sign of vibration reoccuring. In fact the magnets spun like a great top, though the screwed-up bearing permitted no shake of the shaft within it. On passing the critical speed when stopping, the oscillation set up is hardly noticeable.

Before beginning the experiments it was necessary to determine what effect, if any, the magnets had on the vessel A. For this purpose it was filled with distilled water, and the magnets made to run at speeds high and low; but not the slightest movement of the scale could be detected by the microscope so long as the magnets were revolving. Had it moved the hundredth part of a millimetre, that must have been seen ; but when the magnets were stopped the scale was found to have moved; and as the magnets were turned round to successive positions through 360°, the scale went through a complete oscillation of 18 divisions. These two preliminary experiments showed that, though the cell was affected by placing the magnet in different positions (an effect probably due to the three turns of the torsion-wire round the reel in the beam having a directive action), yet electrically the cell was all that could be desired, and any torsion observed during the rapid revolution of the magnets must be due to the conductivity of the liquid in the cell. No currents could be induced in the torsion-wire round the reel, for they formed an open circuit ; but even if closed, the reaction of the induced currents on the magnets would be inappreciable, owing to the small diameter of the coil and its great distance above the magnets. It is also clear that the magnets could have no mechanical action on the cell due to air-currents, vibration, &c. It should be mentioned here that the screen was fixed to a panel screwed to the wall, so as to be quite independent of any thing connected with the magnets; also the torsion-wire was hung from a bracket fixed to the wall under the cornice of the room. This separation of the supports of the various parts is to ensure there being no mechanical action causing torsion due to vibration &c., an effect observed by Snow Harris in certain of his experiments.

Having now made clear the construction of the apparatus, it will be well to explain next the principle of its action, and then to describe the method of working with it, giving at the same time the few results that have been obtained so far.

When a conductor is in a moving magnetic field it is urged in the direction of motion by a force which varies directly as the relative movement, directly as the intensity of the field, and directly as the conductivity of the conductor, *i. e.* if the form of the conductor is always the same. In the present case the form of the conductor is necessarily always the same, and its situation in the magnet is so too; for its height is adjusted by the screw *z* till a horizontal line on the scale *s* is seen on the cross-wires in the microscope. The strength of the magnet is nearly constant; and its variations are allowed for by a method which will be described below: the speed of the magnets can be accurately measured by the counter; and therefore the torsion of the wire *w* gives an exact comparative measure of the conductivity of the liquid in the vessel A, provided that the torsion of the wire is uniform and that the movement of the liquid in the cell has no influence on the result. As the wire used cannot be made to carry 7 lbs., but has in the course of the experiments to support the cell together with a litre of liquid of any specific gravity between those of water and oil of vitriol, its torsion is any thing but constant from day to day; but its variations, due to these great changes in the load and to slight changes in temperature, are accurately allowed for at the same time that the change in the strength of the magnet is taken into account; and therefore, if movement of liquid in the cell does not vitiate the results, this method of comparing the conductivity of electrolytes is free from error.

To ascertain what effect movement of liquid in the cell has on the result, it will be necessary to examine more closely how the torsion of the wire is produced. When the magnets are revolving, currents are induced in the liquid in a direction to oppose the motion; so the liquid is pushed round in the direction in which the magnets are revolving by a force which is directly proportional to the difference in speed between the liquid and the magnets. If there were no friction of any kind tending to resist the motion of the liquid, it would in time attain a velocity equal to that of the magnets; for so long as it was revolving more slowly than the magnets there would be a force urging it on. But there is friction between the outer layer of the liquid and the vessel, and between each successive cylindrical layer and the one outside it; and

therefore the liquid never attains any great velocity at all. But each elementary cylindrical layer soon reaches that speed at which the force urging it forward is equal to the friction between it and the layer outside; and therefore the torsion measured is exactly the same that would be obtained if it were possible to integrate the force from the centre to the edge of the vessel. If this reasoning is correct, the liquid should be revolving fastest at the centre and more slowly outside; and this has been proved to be the case by a very striking experiment. A solution of indigo was made of such a specific gravity that a drop of it would just sink in the liquid used; and when the magnets were revolving at a great speed a series of drops were delivered at different distances from the centre. They all fell in spiral lines, each one revolving more quickly than the one outside it. Then, on reversing the engine and repeating the experiment the spirals were found to be going the other way, but, in each case, in the same direction as the magnets. The average speed of the liquid (30 p. c. H_2SO_4, 70 p. c. H_2O) was about one turn in ten minutes. It is now possible to see to what extent movement of liquid in the cell affects the result. As soon as the rotation of the liquid has become constant, the force urging it forward is equal to that due to friction retarding it; the torsion of the wire therefore is an exact measure of the force on the *moving* liquid. But the force is directly proportional to the relative speed, and not to the actual speed, of the magnets; and we have seen that the liquid does not revolve more than once for 20,000 turns of the magnets; and therefore the error made on the supposition that the liquid is at rest is not more than the 20,000th part of the whole result—a quantity altogether inappreciable, for neither the speed nor the torsion can be measured with such accuracy.

The apparatus described was finished and the experiments begun on April 5, 1880; but it was immediately found that the behaviour of the wire was such that no results of any value could be obtained; for the zero on the scale (that is, the position without torsion) was constantly changing, so much so that sometimes without any apparent cause the scale would move ten divisions in a few hours. As 11·4 divisions correspond to an angular movement of one degree, and as the greatest torsion ever observed caused a movement of only 45

divisions, it seemed that some other wire would have to be used. But, again, steel was the only metal that could be used, as a weaker metal must have been much thicker, and the torsion of a thicker wire would have been too great. Weights were hung on the wire and twisted several times round and then left, to see if the wire would improve by such treatment, but without effect. Then some other wire was sent for, and the machine left for six weeks. During this time the old wire became a little rusty; and when the experiments were to have been continued with the new wire, the old wire was tried once more and was found greatly improved, possibly from the removal of the hard skin by rust. During the course of one experiment, lasting over an hour, the zero had not shifted the tenth part of a division. The next day the same acid was examined again; and the two results differed by only 1·3 p. c. This result seemed very satisfactory, especially as possible changes in the strength of the magnet or in the torsion of the wire were not considered; and therefore we determined that the experiments should be continued with the old wire. So it was well rubbed with sperm oil to stop the rust from increasing in quantity and destroying the wire.

The method of carrying out the experiments must be described next. It has already been shown that the magnets when at rest have a directive action on the cell, but when in motion their action on the cell itself is nothing; yet if a conducting liquid is in it the cell is turned round. Also the zero of the scale can only be determined after the liquid is put in; and therefore there is no possibility of observing the zero directly. But as the torsional effect is proportional to the speed, by running the magnets first at a low and then at a high speed and taking the difference in the speeds (S) and also the difference in the readings (T), and dividing S by T, a number is obtained which is a measure of the resistance of the liquid in the cell. To diminish as much as possible all chances of error, every liquid was examined at four speed — two with the magnets turning in one direction (+), and two in the other direction (—); and the results were taken in pairs, in the order of observation, to avoid errors due to such slow changes in the zero as were still liable to take place. If the strength of the magnets and the torsion of the wire were both

constant, nothing more would be required; but as both are liable to change, some standard of comparison by which the value of an observed deflection can be measured is necessary. The most obvious plan is occasionally to examine some given liquid of good conductivity, *e.g.* 30 p. c. sulphuric acid, and consider variations in the result due to changes in the machine, and correct the measures obtained for other liquids accordingly. But this plan would involve a great waste of time, as from two to three hours are required for the examination of a liquid; and it would not be trustworthy, for it cannot be supposed that the torsion of a wire is the same when stretched to 50 p. c. and to 80 p. c. of its breaking strain; and therefore the standard of comparison must be applied at every experiment when the liquid under observation is in the cell. Accordingly a disk of thin sheet brass, D, three inches in diameter, was used as a comparison plate. It is hung to the glass hook below the beam B, immediately after the observation at the fourth speed has been made, without stopping the engine or touching any other part of the apparatus, and the increase in torsion noted. This increase in the torsion when corrected for speed should be constant; and the slight variations in the increase are due to changes in the magnets, or the wire, or both, the effect of which may therefore be allowed for accurately. It is true that possible changes in the distribution of the magnetism cannot be taken in to account; so that, if the upper magnets changed more or less than the lower ones, the comparison plate would indicate too great or too small a change; but no error of any importance is likely to result from such a cause. Also changes in the conductivity of the brass due to temperature have not been considered, though they can be at any time. After the observed conductivity of a liquid has been corrected by means of this comparison plate, it is independent of changes in the magnets or the wire, and so all results obtained during any length of time are directly comparable. Of course, after any great length of time an experiment would be made on 30-p.-c. sulphuric acid to serve merely as a check. There is one more point that must be noticed before giving the results. Owing to the high specific gravity of sulphuric acid, it seemed hardly safe to use the full quantity of 1000 c. c. of acids stronger than 60 p. c.; and therefore the smaller quantity of 750 c. c.

was used; and in order to compare the effect produced by 750 c. c. with that by 1000 c. c., 750 c. c. of 35-p.-c. acid was taken, and its effect observed after correction by the comparison plate. The apparent resistance obtained was 1·66 times the true resistance given by 1000 c. c.; and therefore, of the four solutions containing respectively 70, 80, 90, and 95·5 p. c. anhydrous sulphuric acid (H_2SO_4), only 750 c. c. were taken, and the observed resistance divided by 1·66.

The method of calculating the results is given next. The speed is measured by the number of turns in one second. Each of these numbers, together with the reading of the scale

H_2SO_4 25; H_2O 75. T. 16° C.					
Exp.	*a.*	*b.*	*c.*	*d.*	*e.*
Speed	−16·81	−34·60	+31·59	+14·68	+14·04
Reading	108·7	88·1	165·3	145·8	174·3

in the line below, is the result of at least four observations. *e* is taken with the comparison plate attached. The first four columns are then taken in pairs, and the sum or the difference of the two speeds taken, according as they are of unlike or of like sign, and put in the first column of the next table; then the differences in the readings are taken and put in the second column.

	Speed.	Torsion.	Zero.	$\frac{\text{Speed}}{\text{Torsion}}$.
a and *b*	17·79	20·6	128·2	·864
b and *c*	66·19	77·2	128·4	·857 } ·860
c and *d*	16·91	19·5	128·9	·867
e................	14·04	45·6 16·3 29·3	128·7	·4795

$\frac{\cdot 860}{\cdot 4795}=1\cdot 794$, the corrected resistance of the liquid;
55·80, the corrected conductivity.

The zero in the next column is obtained from the two observations by a simple proportion sum; and the numbers in the last column are obtained as shown, and are a measure of the resistance of the liquid, but uncorrected by the comparison plate. The three figures obtained differ; but the greatest weight is given to the observation at the highest speed, and ·860 is taken as a fair mean. To make use of the observation *e*, the probable zero must be found by examining the numbers already obtained. As they indicate a gradual rise in its position, a number higher than the mean is taken. This shifting in the zero caused more uncertainty than any thing else; and it is this that limits the accuracy of the investigation. The probable zero subtracted from the reading 174·3 gives the torsion 45·6, due to the liquid and to the plate together; but the torsion due to the liquid alone, of the apparent resistance ·860 already found, and with the speed 14·04, is 16·3; and this, subtracted from 45·6, leaves 29·3, the torsion due to the plate alone; and this divided into the speed 14·04 gives the quotient ·4795, an arbitrary number, the changes in which from time to time represent changes in the magnets or wire. From this the corrected resistance 1·794 is found as shown; and its reciprocal is the conductivity. All these numbers are in themselves entirely arbitrary; but they are all comparable with one another; so that any number of liquids may have their conductivities compared with one another, and eventually with that of some standard metal.

The conductivities of sulphuric acid and of sulphate of copper, the only liquids examined at present, are given in the following Table. The conductivity-curve of sulphuric acid agrees in every important particular with Kohlrausch's, which was obtained with alternating currents, the chief difference being rather a sharper rise to and fall from the first maximum; otherwise the position of the two maxima, of the minimum, and of the point of contrary flexure agree most perfectly*. The experiments will be continued on other liquids.

* A large quantity of oil of vitriol was boiled in platinum capsules, cooled, and thoroughly mixed. 3·7496 grams gave 8·5114 grams Ba_2SO_4, showing 95·5 per cent. of H_2SO_4. After dilution to obtain the requisite strengths, the clear acids were siphoned off from the deposited sulphate of lead.

Per cent.*	Conductivity of			
	H_2SO_4.	T.	$CuSO_4$.	T.
		°C		°C.
5	18·53	... 19	1·62	... 17
10	34·10	... 17	2·56	... 17
15	......	... ...	3·58	... 17·5
17	......	... ...	4·47	... 19
20	47·72	... 17		
25	55·80	... 16		
30	63·22	... 16		
35	58·99	... 15		
40	54·01	... 15·5		
50	48·23	... 16		
60	31·26	... 16·5		
70	20·34	... 16		
80	11·61	... 16·3		
90	10·06	... 18		
95·5	10·74	... 18		

* *Conductivity of Sulphuric Acid and Water.*

Abscissæ are percentages of H_2SO_4. *Ordinates* are conductivities.